Rain and Lightning

Task

a. Today there is a 55% chance of rain, a 20% chance of lightning, and a 15% chance of lightning and rain together. Are the two events "rain today" and "lightning today" independent events? Justify your answer.

b. Now suppose that today there is a 60% chance of rain, a 15% chance of lightning, and a 20% chance of lightning if it's raining. What is the chance of both rain and lightning today?

c. Now suppose that today there is a 55% chance of rain, a 20% chance of lightning, and a 15% chance of lightning and rain. What is the chance that we will have rain or lightning today?

d. Now suppose that today there is a 50% chance of rain, a 60% chance of rain or lightning, and a 15% chance of rain and lightning. What is the chance that we will have lightning today?

Commentary

This task uses the same situation to explore different concepts of probability theory.

Part (a) explores the idea of independence of events. Students must use the fact that two events A and B are independent if $P(A \text{ and } B) = P(A) \cdot P(B)$.

Part (b) lets students explore the idea of conditional probability. Students should understand the difference between $P(A \text{ and } B)$ and $P(A|B)$. They should also understand that $P(A \text{ and } B) = P(B \text{ and } A)$. In this part, they are given three probabilities, but only two of them are needed to answer the question. In general, students very often assume that every piece of given information must be used in the solution, so this is a good way to realize that sometimes we have more data than we really need, and have to choose which information is useful.

In part (c) students practice using the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. Teachers should make students aware of the fact, that in mathematics the expression "or" is almost always used in the inclusive sense, that is "A or B" really means "either A or B or both". Students must also understand that $P(A \text{ or } B) = P(B \text{ or } A)$ and that $P(A \text{ and } B) = P(B \text{ and } A)$.

Part (d) is a variant of part (c). Students must apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and solve the equation for the unknown quantity.

Solution

a. Given
- $P(\text{rain}) = .55$,
- $P(\text{lightning}) = .2$, and
- $P(\text{lightning and rain}) = .15$.

Two events are independent if

$$P(\text{lightning and rain}) = P(\text{lightning}) \cdot P(\text{rain}).$$

Since

$$P(\text{lightning}) \cdot P(\text{rain}) = (.55) \cdot (.2) = .11 \neq .15 = P(\text{ lightning and rain})$$

the two events are not independent.

b. Given
- $P(\text{rain}) = .6$,
- $P(\text{lightning}) = .15$, and
- $P(\text{lightning} \mid \text{rain}) = .2$.

We need to find $P(\text{rain and lightning})$. We use the formula

$$P(\text{lightning} \mid \text{rain}) = \frac{P(\text{lightning and rain})}{P(\text{rain})}.$$

Since we have two of the three pieces of information, we have to solve for the third one. Multiplying both sides of the equation by P(rain) we get

$$P(\text{lightning and rain}) = P(\text{lightning} \mid \text{rain}) \cdot P(\text{rain}) = (.2) \cdot (.6) = .12$$

Answer: There is a 12% chance of both rain and lightning today.

c. Given
- $P(\text{rain}) = .55$,
- $P(\text{lightning}) = .2$, and
- $P(\text{lightning and rain}) = .15$.

We need to find $P(\text{rain or lightning})$, which is the same as $P(\text{lightning or rain})$. Using the Addition Rule we obtain

$$P(\text{lightning or rain}) = P(\text{lightning}) + P(\text{rain}) - P(\text{lightning and rain}) = .2 + .55 - .15 = .6$$

Answer: There is a 60% chance of rain or lightning today.

d. Given
   - $P(\text{rain}) = .5$,
   - $P(\text{lightning}) = .6$, and
   - $P(\text{lightning and rain}) = .15$.

   We need to find P(lightning). We use the Addition Rule:

   $$P(\text{rain or lightning}) = P(\text{rain}) + P(\text{lightning}) - P(\text{rain and lightning}).$$

   Since we have three of the four pieces of information, we have to solve for the fourth one, the probability of lightning. Subtracting $P(\text{rain})$ and adding $P(\text{rain and lightning})$ to both sides of the equation, we obtain

$$P(\text{lightning}) = P(\text{rain or lightning}) - P(\text{rain}) + P(\text{rain and lightning}) = .6 - .5 + 15 = .25$$

   Answer: There is a 25% chance of lightning today.