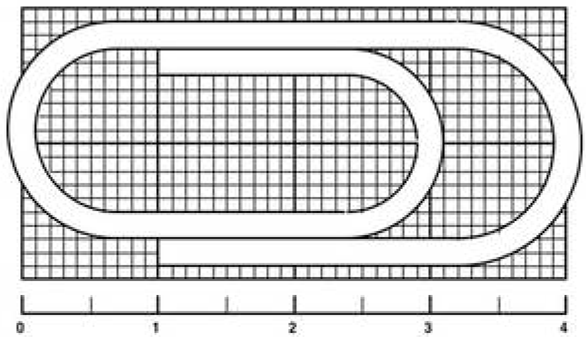
Paper Clip

Task

This paper clip is just over 4 cm long.



How many paper clips like this may be made from a straight piece of wire 10 meters long?

Commentary

This high level task is an example of applying geometric methods to solve design problems and satisfy physical constraints. This task is accessible to all students. In this task, a typographic grid system serves as the background for a standard paper clip. A metric measurement scale is drawn across the bottom of the grid and the paper clip extends in both directions slightly beyond the grid. Students are given the approximate length of the paper clip and determine the number of like paper clips made from a given length of wire. Extending the paper clip beyond the grid provides an opportunity to include an estimation component in the problem. In the interest of open-ended problem solving, no scaffolding or additional questions are posed in this task. The paper clip modeled in this problem is an actual large standard paper clip.

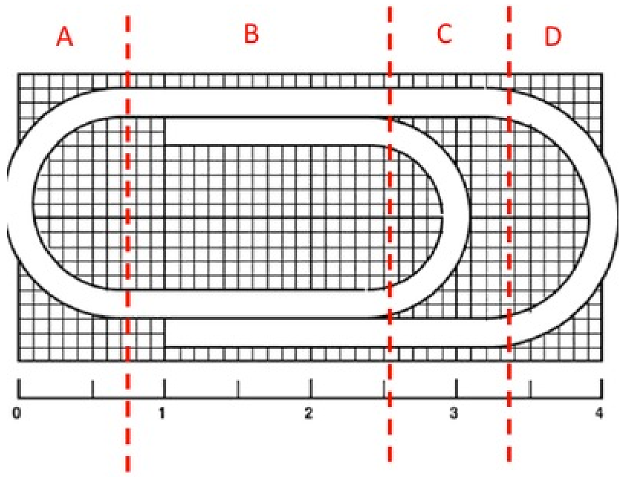
This open-ended task has a multitude of entry levels. Students will apply prior knowledge to solve this problem and their understanding of concepts such as dividing a composite figure into familiar geometric representations, circumference of a circle, circumference of a semi-circle, symmetry, estimation, and metric measurement will determine their entry level and solution path of choice. Many different solution paths are possible and should not be limited to the solutions provided. Given the estimation component built into this problem, we encourage you to accept all reasonable responses. Materials made available for open-ended problem solving increases the likelihood of varied responses.

Suggested Materials:

- String or thin picture wire
- Straightedge

Solutions

Solution: Solution 1
Basic Level Responses
One approach is to divide the paper clip into vertical regions, and then to use the measurement grid to determine the length of the straight sections and estimate the length of the curved sections using a string or thin wire in conjunction with the measurement scale provided. One such division is accomplished using three vertical dividers splitting the paper clip into four distinct regions as shown.



Regions	Number of Linear Sections	Number of Curved Sections
Region A	0	1
Region B	4	0
Region C	2	1
Region D	0	1

The lengths of the linear sections were determined using the gridlines. The estimations of the lengths of the curved sections were determined using a string or thin wire in conjunction with the measurement scale provided.

Regions	Measurement of Linear Sections (listed from top to bottom)	Estimated Measurement of Curved Sections
Region A		2.5cm
Region B	1.8cm, 1.5cm, 1.8cm, 1.5cm	
Region C	0.8cm, 0.8cm	1.6cm
Region D		2.5cm

The length of wire needed to manufacture one paper clip is now approximately:

$$1.8\text{ cm} + 1.5\text{ cm} + 1.8\text{ cm} + 1.5\text{ cm} + 0.8\text{ cm} + 0.8\text{ cm} + 2.5\text{ cm} + 1.6\text{ cm} + 2.5\text{ cm} = 14.8\text{ cm}$$

The length of the straight piece of wire is 10 meters. Since 1 meter is the same as 100 centimeters, 10 meters is $10 \cdot 100 = 1000$ centimeters. Finally, we find that at 14.8 cm per paper clip, 1000 centimeters will produce approximately

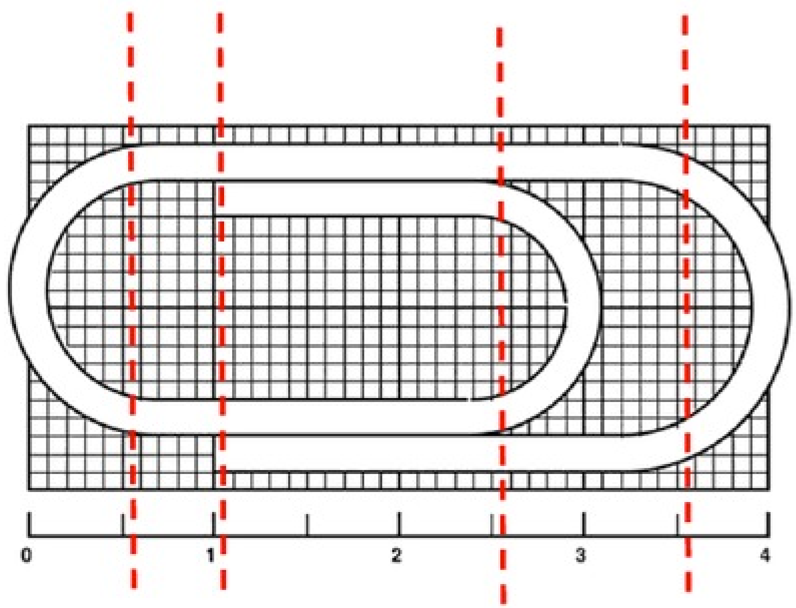
$$\frac{1000}{14.8} \approx 67.6 \text{ paper clips.}$$

Since we can only make a whole number of paper clips, we conclude that approximately 67 paper clips may be manufactured from a straight piece of wire 10 meters in length.

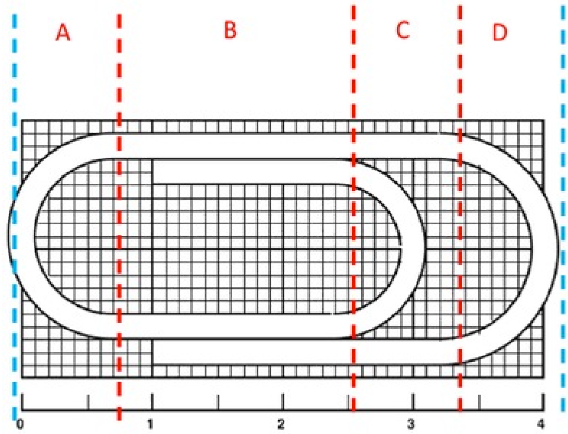
Note: Students may or may not notice the two different units of measure used in this task. The paper clip described in terms of centimeters, and the piece of wire is described in terms of meters. This will be apparent if 10 meters is not converted to 1,000 centimeters when determining the number of paper clips that can be manufactured. The use of string to estimate the length of the curved sections of the paper clip creates a less than accurate outcome. There is a difference in the length of the curved section in region A and the curved section in region B. This difference is not always obvious when using a string measurement strategy because the difference in the length of the curved sections is small (0.2cm). An additional source for miscalculation is where students actually place the string when attempting to trace the curvatures. Each curved section has an outer and inner radius due to the thickness of the wire which may lead to potential overestimation or underestimation.

Alternate Strategy Used to Divide the Paper Clip into Regions

This strategy uses only designated tick marks on the measurement scale and divides the paper clip vertically into five regions while isolating the curved portions.



Solution: Solution 2
Advanced Level Responses
One approach is to divide the paper clip into vertical regions, use the measurement grid to determine the length of the straight sections and use geometric formulas to determine the length of the curved sections. This division can be accomplished, for example, by using three vertical dividers splitting the paper clip into four distinct regions as shown. The outer two additional lines extend the grid and serve as boundaries for the two outer regions. These additional lines aid in the estimation of the length of the paper clip drawn beyond the gridlines.



The vertical division lines were purposely placed such that each curved section of the paper clip appears to be in the shape of a semi-circle. The radius of the semi-circle is determined using the gridlines and the gridline extension to accommodate the portion of the paper clip beyond the grid. The estimated lengths of the curved sections were determined using the semi-circumference formula, $C = \pi r$. There is, however the same concern as in the previous solution; namely, deciding whether to use the outer radius, the inner radius, or a radius somewhere between the two. The below table shows the results of using in each case a radius very close to the outer radius.

Regions	Number of Curved Sections	Length of Radius in Curved Sections	Estimated Circumference of Circle	Estimated Circumference of Semi-Circle
Region A	1	$r = 0.8$ cm	5.0 cm	2.5 cm
Region B	0			
Region C	1	$r = 0.6$ cm	3.8 cm	1.9 cm
Region D	1	$r = 0.9$ cm	5.6 cm	2.8 cm

The lengths of the linear sections were determined using the gridlines.

Regions	Number of Linear Sections	Measurement of Linear Sections (listed from top to bottom)
Region A	0	
Region B	4	1.8cm, 1.5cm, 1.8cm, 1.5cm
Region C	2	0.8cm, 0.8cm
Region D	0	

The length of wire needed to manufacture one paper clip:

$$1.8\text{ cm} + 1.5\text{ cm} + 1.8\text{ cm} + 1.5\text{ cm} + 0.8\text{ cm} + 0.8\text{ cm} + 2.5\text{ cm} + 1.9\text{ cm} + 2.8\text{ cm} = 15.4\text{ cm}$$

The length of the straight piece of wire is 10 meters. Since 1 meter is the same as 100 centimeters, 10 meters is $10 \cdot 100 = 1000$ centimeters. Dividing, we find that at 15.4 cm per paper clip, 1000 centimeters will produce approximately

$$\frac{1000}{15.4} \approx 64.9 \text{ paper clips.}$$

Since we can only make a whole number of paper clips, we conclude that approximately 64 paper clips may be manufactured from a straight piece of wire 10 meters in length.

Note: Using geometric formulas to calculate the curved sections in each region are a bit more dependable than using a string strategy, but accuracy is still compromised. Students may miscount the gridlines when determining the radius of the semi-circle, and there is a rounding off issue when pi is used in the formula to compute the circumference. Then to compound the degree of error, the length of the paper clip which is an approximation is divided into the 10 meter or 1,000 centimeter piece of wire.

Alternate Strategy Used to Divide the Paper Clip into Regions
This strategy divides the paper clip horizontally through the middle into two regions taking advantage of horizontal symmetry. Each section of wire above the line can be mapped onto the exact section of wire below the line. Students calculate lengths associated with the upper or lower half of the paper clip and double the results.