How thick is a soda can II?

Task

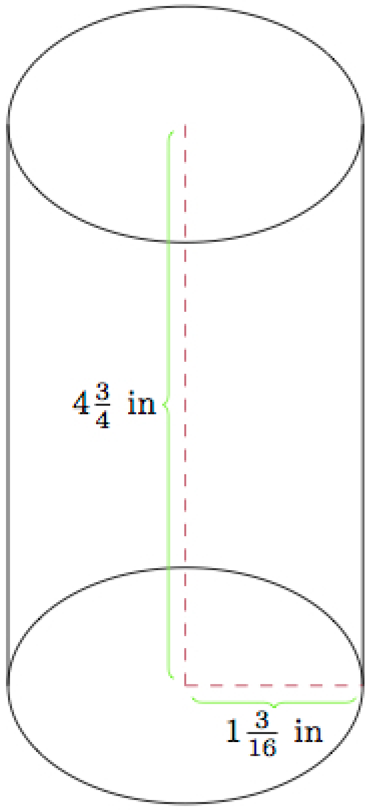
About how thick is a soda can?

Explain which measurements you will need to take as well as extra information that you may need in order to estimate the thickness of a soda can. You may assume that the can is made of aluminum. Because of the risk of injury, cutting the can and directly measuring its thickness is not permitted.

Solutions

Solution: 1 Use density of aluminum

A picture of the soda can is given here. This can be shared with the students if they do not have access to soda cans or to the appropriate measuring equipment that they will need to estimate its thickness.



The only part of an aluminum can which gives us a visual sense for how thick the can might be is the opening from which we drink the soda. Feeling and observing this, it is quite sharp and clearly very thin, certainly too thin to be measured with a normal tape measure. The thickness of the can will, however, be proportional to how much aluminum is in the can and this, in turn, can be captured by weighing the can. In addition to the weight of the can, the students will need to know the density of aluminum (that is, how much does a given amount of aluminum weigh) and they will need to approximate the surface area of the can.

The surface of the can is made up of the top and bottom together with the cylindrical part of the can. In practice, the top and bottom are not flat but are well approximated by circles. According to the given information, the radius of these circles is about $1\frac{3}{16}$ inches. So the area is about $\pi\times\left(1\frac{3}{16}\right)^2$ square inches or about 4.4 square inches. If the cylindrical part of the can were made flat, it would be a rectangle with dimensions $4\frac{3}{4}$ inches by $\pi\times2\times1\frac{3}{16}$ inches or about 35.4 square inches. So the total surface area of the can is about 44.2 square inches.

If there are about 15 grams of aluminum in the soda can and the density of aluminum is about 2.70 grams per cubic centimeters then there are about

$$\frac{15 \text{ grams}}{2.70 \text{ grams per cubic centimeter}} \approx 5.6 \text{ cubic centimeters}$$

of aluminum in the soda can.

Since the amount of aluminum is given in cubic centimeters and the area of the soda can in square inches we need to make a conversion in order to estimate the approximate thickness of the can. There are about 2.54 centimeters per inch and so there are about $(2.54)^2 \approx 6.5$ square centimeters per square inch. So

$$44.2 \text{ square inches} \approx 44.2 \times 6.5 \text{ square centimeters.}$$

This is about 287.3 square centimeters. To find the approximate thickness of the can we know that

$$\text{thickness} \times 287.3 \text{ cm}^2 \approx 5.6 \text{ cm}^3.$$

This means that the approximate thickness of the can is about 0.02 centimeters or 0.2 millimeters.

If the teacher wishes to engage students in a discussion of precision of measurements and how results should be recorded, there are three measurements which are subject to error as well as the reported density of aluminum which is only accurate the nearest one hundredth of a gram per cubic centimeter. The measurements of the can, if made with a tape measure marked to the nearest sixteenth of an inch, can be assumed to be accurate to within $\frac{1}{32}$ of an inch. This is about 2.5 percent of the measurement of the radius and seven tenths of one percent of the measurement for the height. Finding the approximate area of the top and bottom of the can requires squaring the radius which will roughly double the error to 5 percent. We have that 5 percent of 8.8 square inches is almost half a square inch. The error for the cylindrical part of the can will depend on the error for the height and the error for the circumference and will be a little more than 3 percent or a little more than one square inch. So the total possible error here is close to one and a half square inches. An appropriate way to record this would be $44 \pm \frac{3}{2}$ square centimeters and then this error continues through the final estimation of the thickness of the aluminum can. The can is not perfectly cylindrical and it has a tab to open the can: these will contribute to the error as well but the overall estimate of 0.2mm for thickness should be good provided this thickness is uniform.

Solution: Solution 2 Use Archimedes' principle
In order to estimate how thick the soda can is, we can estimate the surface area of the can and the volume of the aluminum in the can and then use the fact that the volume of aluminum in the can is approximately equal to the surface area of the can times its thickness. For the surface area, the method of the first solution can be used and we repeat this here.

The surface of the can is made up of the top and bottom together with the cylindrical part of the can. In practice, the top and bottom are not flat but are well approximated by circles. According to the given information, the radius of these circles is about $1\frac{3}{16}$ inches. So the area is about $\pi \times \left(1\frac{3}{16}\right)^2$ square inches or about 4.4 square inches. If the cylindrical part of the can were made flat, it would be a rectangle with dimensions $4\frac{3}{4}$ inches by $\pi \times 2 \times 1\frac{3}{16}$ inches or about 35.4 square inches. So the total surface area of the can is about 44.2 square inches. There are about 2.54 centimeters per inch and so there are about $(2.54)2 \approx 6.5$ square centimeters per square inch. So

$$44.2 \text{ square inches} \approx 44.2 \times 6.5 \text{ square centimeters.}$$

This is about 287.3 square centimeters.

The volume of the can is not given and so this needs to be assessed by some indirect means. Using a technique dating back to Archimedes, if we submerge the can in water, then the amount of water displaced by the can will be equal to its volume. One way to do this would be to crush the can (although some care is needed here to make sure that in so doing we do not trap too much air within the can). Another method would be to carefully fill the can to the top with water and then the filled can will sink. The amount of water displaced by the aluminum in the can should be close to 5.6 cubic centimeters as was found in the first solution. Experimental values may differ, however, so suppose we find that the aluminum in the soda can displaces x cubic centimeters of water. Then using the equation

$$\text{Volume} \approx \text{surface area} \times \text{thickness}$$

we can plug in x for the volume and 287.3cm^2 for the surface area and we find

$$\text{thickness} \approx \text{x}287.3\text{cm}^2.$$

Notice that this method is quicker than the first and requires less information. On the other hand, it does require some lab equipment.