

How many cells are in the human body?

Task

About how many cells are in the human body?

You can assume that a cell is a sphere with radius 10^{-3}cm and that the density of a cell is approximately the density of water which is 1g/cm^3 .

Commentary

The purpose of this task is for students to apply the concepts of mass, volume, and density in a real-world context. There are several ways one might approach the problem, e.g., by estimating the volume of a person and dividing by the volume of a cell. The main pitfall with that approach is that students generally know how much a person weighs, but are less likely to make accurate estimates of a person's volume. The task provides an opportunity to think about attention to mathematical precision. Note that despite maintaining several digits of accuracy throughout the calculation, we report an answer with only one significant digit.

After students have worked on this problem, teachers should spend some time discussing the reasonableness of the assumptions provided. Cells are not really spherical, for example, but getting the right order of magnitude for the volume is probably sufficient for this type of estimation. (For example, if we replace our spherical model of a cell by a cubical model, our net estimate will be cut approximately in half). Also, different cells are likely to have different densities and are not all packed together as tightly in all parts of the body (consider bone cells, for example). This task could be nicely paired with work in a biology class where students could discuss these issues in more depth.



Solution

First, assume that a cell is roughly a sphere with radius 10^{-3} cm. Let's compute the volume of the cell from the relation between radius and volume:

The volume V of a sphere with radius r is

$$V = \frac{4\pi}{3} r^3$$

where r^3 and V have the same units. The volume of a cell with radius 10^{-3} cm is therefore

$$V = \frac{4\pi}{3} (10^{-3})^3 \approx 4.1888 \times 10^{-9} \text{ cm}^3.$$

Next, we use the fact that the density of a cell is approximately the density of water which is 1 g/cm^3 . Let's compute the mass of a cell from its volume and density using the definition of density:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

which is equivalent to

$$\text{mass} = \text{density} \times \text{volume}.$$

The mass of a cell is therefore

$$1 \frac{\text{g}}{\text{cm}^3} \times 4.1888 \times 10^{-9} \text{ cm}^3 \approx 4.1888 \times 10^{-9} \text{ g}.$$

Now you know the mass of a cell, what other piece of information do you need to work out the number of cells in your body? Your mass in grams (if you know your weight in pounds, you can convert: $1 \text{ lb} \approx 453.6 \text{ grams}$).

$$\text{Number of cells} = \frac{\text{Your Mass}}{\text{Mass of Cell}}$$

For example, the mass of a 100 lb child is about 45,360 grams. We have that

$$\frac{45,360 \text{ g}}{4.1888 \times 10^{-9} \text{ g}} \approx 1.0829 \times 10^{13}$$

so there are approximately $1 \times 10^{13} = 10,000,000,000,000$ or ten trillion cells in the human body!

