Doctor's Appointment

Task

Jared is scheduled for some tests at his doctor's office tomorrow. His doctor has instructed him to drink 3 liters of water today to clear out his system before the tests. Jared forgot to bring his water bottle to work and was left in the unfortunate position of having to use the annoying paper cone cups that are provided by the water dispenser at his workplace. He measures one of these cones and finds it to have a diameter of 7cm and a slant height (measured from the bottom vertex of the cup to any point on the opening) of 9.1cm.

$$\text{Note: } 1 \text{ cm}^3 = 1 \text{ ml}$$

a. How many of these cones of water must Jared drink if he typically fills the cone to within 1cm of the top and he wants to complete his drinking during the work day?

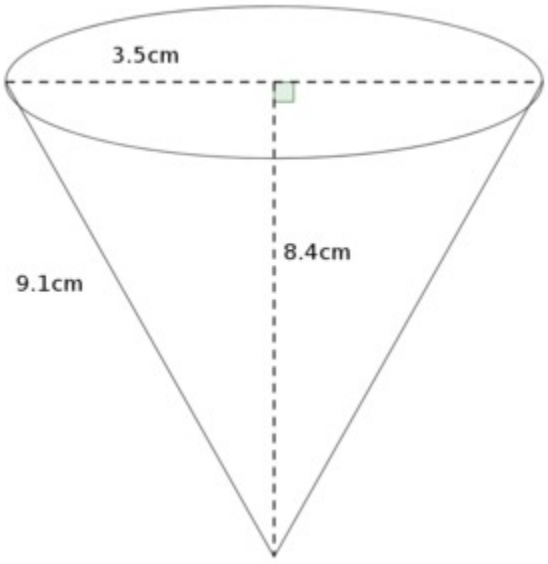
b. Suppose that Jared drinks 25 cones of water during the day. When he gets home he measures one of his cylindrical drinking glasses and finds it to have a diameter of 7cm and a height of 15cm. If he typically fills his glasses to 2cm from the top, about how many glasses of water must he drink before going to bed?

Commentary

The purpose of the task is to analyze a plausible real-life scenario using a geometric model. The task requires knowledge of volume formulas for cylinders and cones, some geometric reasoning involving similar triangles, and pays attention to reasonable approximations and maintaining reasonable levels of accuracy throughout.

Submitted by Patrick Barringer to the Third illustrative Mathematics Task writing contest.

Solution



a. Since the slant height and radius are known (9.1 cm and 3.5 cm, respectively), we can use the Pythagorean Theorem to find the height of the cone as displayed in the equation. (Solving $x^2 + 3.5^2 = 9.1^2$ gives $x = 8.4$). Since we leave the top centimeter of the height of the cup empty, the height of the filled portion would then be 7.4cm. Using similar triangles we find that the radius of the filled area is in ratio $7.4 : 8.4$ to the original radius of 3.5cm, i.e., the radius of the filled region is $\frac{7.4}{8.4} \cdot 3.5 = \frac{37}{12} \approx 3.08$ cm. Using the formula for the volume of a cone we find that each cone of water contains approximately

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3.08)^2(7.4) \approx 73.51$$

cubic centimeters (i.e., millileters) of water. As Jared needs to drink 3000ml of water, dividing $\frac{3000}{73.51} \approx 40.81$ tells us that he needs to drink about 41 cones of water over the course of the day.

b. Multiplying the 25 cones by 73.51ml we find that Jared has already drunk 1837.75ml of his required 3 liters, leaving him with 1162.25ml to still drink. We quickly calculate from the volume of a cylinder that each cupful of his drinking glasses can hold approximately $\pi r^2 h = \pi(13)(3.5)^2 \approx 500$ ml, so he needs to drink just over 2 glasses of water before going to bed.