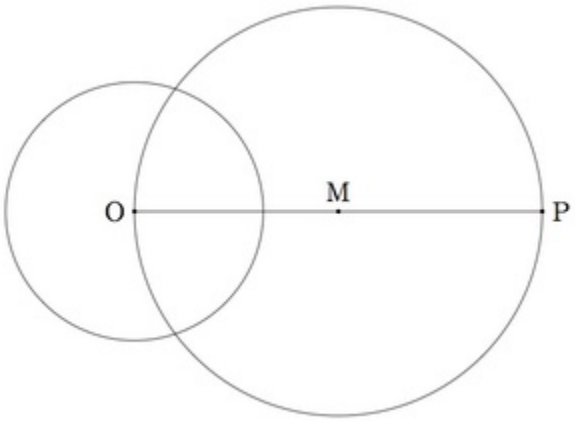
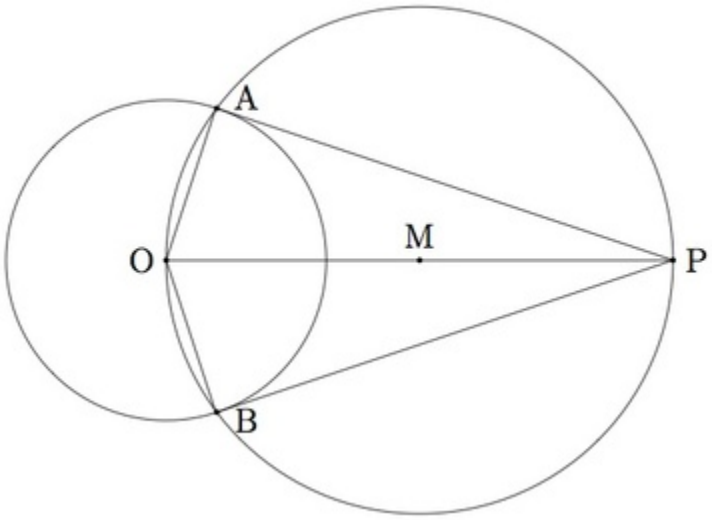
Tangent to a circle from a point

Task

Suppose is a circle with center and is a point our side of . Let be the midpoint of segment and let be the circle with center passing through .



Let and be the two points of intersection of and , pictured below along with several line segments of interest:



a. Show that angles and are right angles.

b. Show that and are tangent lines from to the circle .

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Commentary

The construction of the tangent line to a circle from a point outside of the circle requires knowledge of a couple of facts about circles and triangles. First, students must know, for part (a), that a triangle inscribed in a circle with one side a diameter is a right triangle. This material is presented in the tasks “Right triangles inscribed in circles I.” For part (b) students must know that the tangent line to a circle at a point is characterized by meeting the radius of the circle at that point in a right angle: more about this can be found in ''Tangent lines and the radius of a circle.''

Because of all of the prerequisites this is an ideal task for using some of these other constructions and it serves to reinforce their importance. It might be appropriate to study this problem in order to motivate the constructions required in the solution. Then students can return to this afterward as a reward for having mastered those techniques.

While the solution to this problem appears to be very short this is misleading as each step requires careful attention. One of appealing aspects of geometry and mathematics in general is the way results build upon one another. It is vital that students see some of the more creative and sophisticated constructions like the one performed here in order to feel motivated to learn the supporting structures which make this possible.

If the distance from to is equal to then lies on the circle and so the tangent line from to is the tangent line of at . If the distance from to is less than then lies inside and there is no tangent line from to .

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Solution

Below is a picture of the different points and triangles used in the solution of the problem:

a. Segment is a diameter of circle since the center of is the midpoint of this segment. The points and are also both on since they are the points of intersection of and . The angles and are both right angles because is a diameter of circle and and are points on : this means that triangles and are inscribed in circle and so the angle opposite the diameter must be a right angle.

b. Since angles and are right angles it follows that meets the radius in a right angle and similarly meets radius in a right angle. This means that is tangent to at and is tangent to at .

   Below is a picture with the two tangent lines constructed above:

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