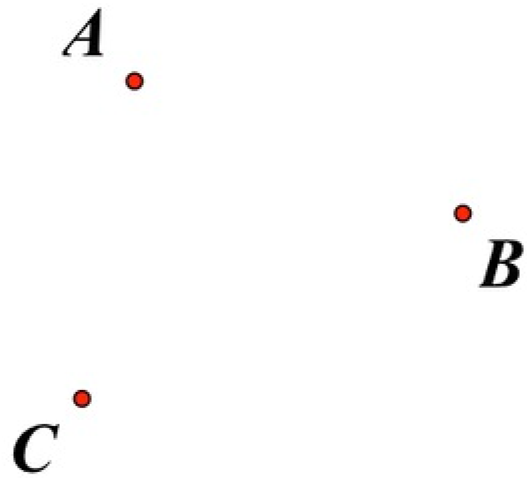
Placing a Fire Hydrant

Task
You have been asked to place a fire hydrant so that it is an equal distance form three locations indicated on the following map.



a. Show how to fold your paper to physically construct this point as an intersection of two creases.

b. Explain why the above construction works, and in particular why you only needed to make two creases.

Commentary

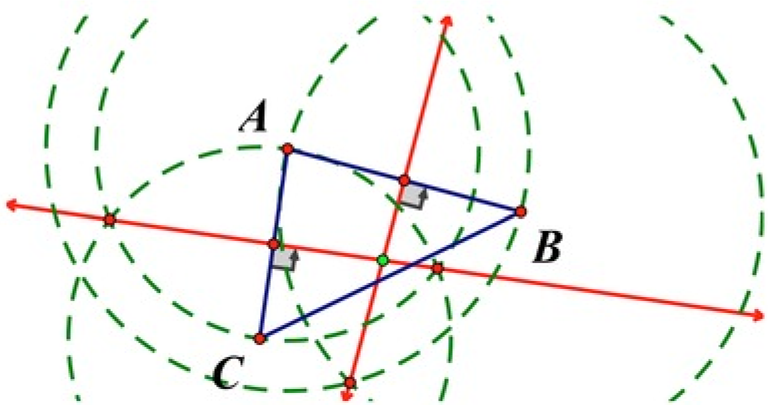
This task can be implemented in a variety of ways. For a class with previous exposure to properties of perpendicular bisectors, part (a) could be a quick exercise in geometric constructions, and an application of the result. Alternatively, this could be part of an introduction to perpendicular bisectors, culminating in a full proof that the three perpendicular bisectors are concurrent at the circumcenter of the triangle, an essentially complete proof of which is found in the solution below.

We note also that the geometric construction aspect of the proof could be nicely replaced with an exploration involving the use of dynamic geometry software.

Solution

a. Fold and crease the paper so that line segment point A lands onto point B. Do the same so that point A lands on point C. The intersection of the two creases is the point we want.

b. Since the desired location is an equal distance from three non-collinear points, we are looking for the center of the circle passing through these three points. This corresponds to the center of the circle circumscribed about the triangle ABC. The center of the circumcircle, called the circumcenter, can be found by constructing the perpendicular bisectors of the three sides of the triangle (precisely the creases made in the paper on the previous step). Since the perpendicular bisectors are concurrent, it is sufficient to construct only two of the three perpendicular bisectors.



The concurrency of the perpendicular bisectors can be argued as follows: Let P be the green dot in the above diagram, the intersection of the perpendicular bisectors of AB and AC. By virtue of P being on the perpendicular bisector of AB, P is equidistant from A and B, i.e., $PA = PB$. Similarly, by virtue of being on the perpendicular bisector of AC, we have $PA = PC$. But this implies that $PB = PC$, i.e., that P is also on the perpendicular bisector of BC, demonstrating that P indeed lies on all three perpendicular bisectors.