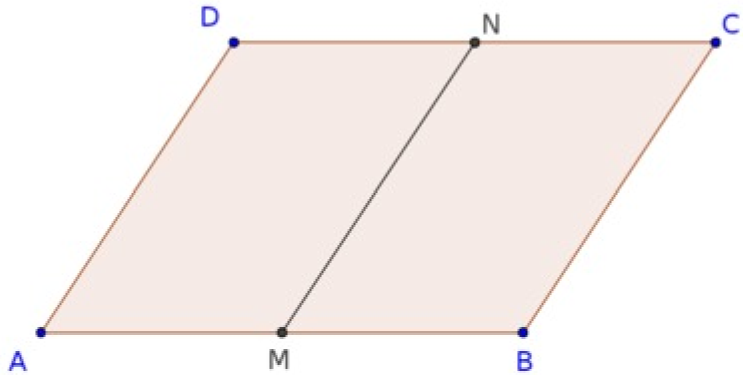
Midpoints of the Sides of a Parallelogram

Task

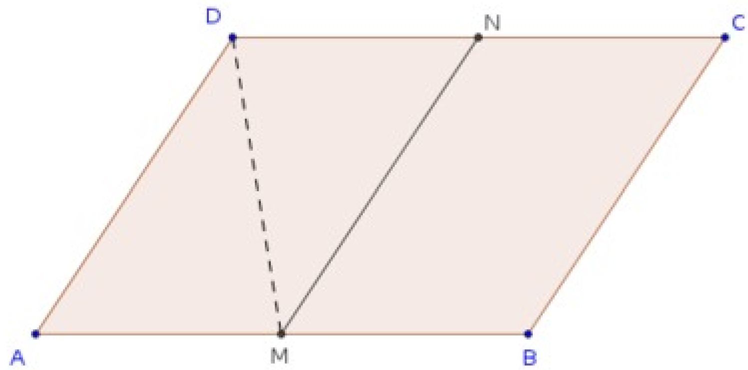
Suppose that $ABCD$ is a parallelogram, and that M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$, respectively. Prove that $MN = AD$, and that the line $\overleftrightarrow{MN}$ is parallel to $\overleftrightarrow{AD}$.



Commentary

This is a reasonably direct task aimed at having students use previously-derived results to learn new facts about parallelograms, as opposed to deriving them from first principles. The solution provided (among other possibilities) uses the SAS trial congruence theorem, and the fact that opposite sides of parallelograms are congruent.

Solution



The diagram above consists of the given information, and one additional line segment, $\overline{MD}$, which we will use to demonstrate the result. We claim that triangles $\triangle\ AMD$ and $\triangle\ NDM$ are congruent by SAS:

a. We have $\overline{MD} = \overline{DM}$ by reflexivity.

b. We have $\angle AMD = \angle NDM$ since they are opposite interior angles of the transversal MD through parallel lines AB and CD.

c. We have $\overline{MA} = \overline{ND}$, since M and N are midpoints of their respective sides, and opposite sides of parallograms are congruent:

$$\overline{MA} = \frac{1}{2}(\overline{AB}) = \frac{1}{2}(\overline{CD}) = \overline{ND}.$$

Now since corresponding parts of congruent triangles are congruent, we have $DA = NM$, as desired. Similarly, we have congruent opposite interior angles $\angle DMN \cong \angle MDA$, so $\overleftrightarrow{MN}$ is parallel to $\overleftrightarrow{AD}$.