US Population 1790-1860

Task

Year	Population (*in thousands*)	Change in Population (*in thousands*)	Successive Population Quotients
1790	3929	$----$	$----$
1800	5308	$5308 - 3929 = 1379$	$\frac{5308}{3929} \approx 1.351$
1810	7240	$7240 - 5308 = 1932$	$\frac{7240}{5308} \approx 1.364$
1820	9638		
1830	12,866		
1840	17,069		
1850	23,192		
1860	31,443		

Source: http://en.wikipedia.org/wiki/Demographic_history_of_the_United_States#Historical_population

a. Complete the table. In the fourth column, round to the thousandths place.

b. Would a linear function be an appropriate model for the relationship between the U.S. population and the year? Explain why or why not.

c. Would an exponential function be an appropriate model for the relationship between the U.S. population and the year? Explain why or why not.

d. Heather decides to use an exponential function of the form $y = a \cdot b^x$ to model the relationship. She chooses 1.359 for the value of b. What meaning does this value have in the context of these data?

e. Use Heather's base value and the population in 1860 to predict the U.S. population in the year 1900.

Commentary

The purpose of this task is to help students learn that exponential functions are characterized by equal growth factors over equal intervals, and that the growth factor over a unit interval is the base b when the exponential function is expressed in the form $f(x) = ab^x$. This task can be used alongside “Equal Factors over Equal Intervals”.

The value 1.359 was chosen because it is the average of the values in the fourth column of the table.

Solution

Year	Population (*in thousands*)	Change in Population (*in thousands*)	Successive Population Quotients
1790	3929	$----$	$----$
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1810	7240	$7240 - 5308 = 1932$	$\frac{7240}{5308} \approx 1.364$
1820	9638	2398	1.331
1830	12,866	3228	1.335
1840	17,069	4203	1.327
1850	23,192	6123	1.457
1860	31,443	8251	1.348

a. No, because the population does not increase by approximately the same amount each decade over the period of time shown in the table.

b. Yes, the table shows that over ten year periods, the population increases by approximately the same factor (about 1.34 per decade). Hence an exponential function is appropriate to model the relationship between the population and the year.

c. The base b should approximate the constant factor by which the population increases each decade. A value of 1.359 would mean that the population is growing by a factor of $1.359 = 1 + 0.359$ or 35.9% per decade. (Note to the teacher: 1.359 is the average of the rounded successive population quotients. The quotients were rounded to the nearest thousandths place.)

d. The population in 1860 was 31,443,000. Heather's choice for the base in problem c) predicts the population grew by a factor of $1.359^4 \approx 3.411$ between 1860 and 1900. Therefore, Heather's model predicts the 1900 population to be approximately $31{,}443 * 1.359^4 \approx 107{,}251$ thousand people.