

Linear or exponential?

Task

In (a)–(e), say whether the quantity is changing in a linear or exponential fashion.

- a. A savings account, which earns no interest, receives a deposit of \$723 per month.
- b. The value of a machine depreciates by 17% per year.
- c. Every week, $\frac{9}{10}$ of a radioactive substance remains from the beginning of the week.
- d. A liter of water evaporates from a swimming pool every day.
- e. Every 124 minutes, $\frac{1}{2}$ of a drug dosage remains in the body.

Commentary

This task gives a variety of real-life contexts which could be modeled by a linear or exponential function. The key distinguishing feature between the two is whether the change by equal factors over equal intervals (exponential functions), or by a constant increase per unit interval (linear functions). The task could either be used as an assessment problem on this distinction, or used as an introduction to the differences between these very important classes of functions.



Solution

- a. Assuming no money is being taken out of the account, the account is increasing in a linear fashion because the same amount is added to the account every month.
- b. The value of the machine decreases by 17% per year. This is the same as saying that the amount, B , gets replaced by $B - 0.17B = 0.83B$. So the value is multiplied every year by a constant factor that is less than 1. Therefore it is decreasing exponentially.
- c. Each week the quantity of radioactive substance gets multiplied by $\frac{9}{10}$, so it is decreasing exponentially.
- d. Every day the amount of water in the pool decreases by the same amount, one liter, so it decreases in a linear fashion.
- e. Every 124 minutes the amount of the drug gets multiplied by $\frac{1}{2}$, so this quantity decreases exponentially.

