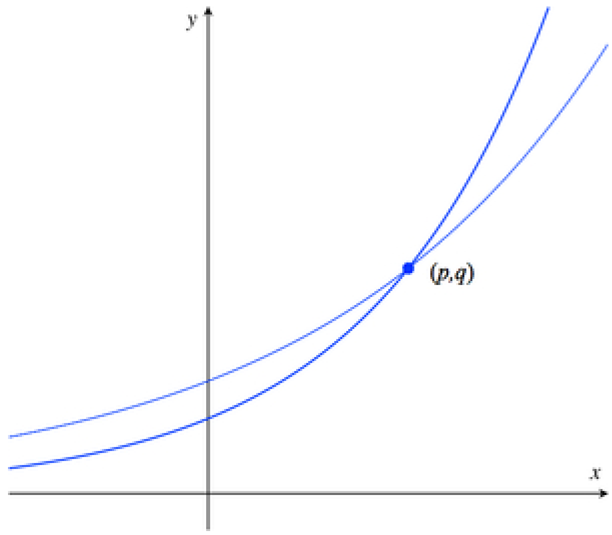
Exponential Functions

Task

The figure below shows the graphs of the exponential functions $f(x) = c \cdot 3^x$ and $g(x) = d \cdot 2^x$, for some numbers $c > 0$ and $d > 0$. They intersect at the point $(p,\ q)$.



a. Which is greater, c or d? Explain how you know.

b. Imagine you place the tip of your pencil at $(p,\ q)$ and trace the graph of g out to the point with x-coordinate $p + 2$. Imagine I do the same on the graph of f. What will be the ratio of the y-coordinate of my ending point to the y-coordinate of yours?

Commentary

This task requires students to use the fact that the value of an exponential function $f(x) = a \cdot b^x$ increases by a multiplicative factor of b when x increases by one. It intentionally omits specific values for c and d in order to encourage students to use this fact instead of computing the point of intersection, (p, q), and then computing function values to answer the question.

This task is preparatory for standard MAFS.912.F-LE.1.1a.

Solutions

Solution: Exponential Functions

a. The graph of $f(x) = c \cdot 3^x$ is steeper than the graph of $g(x) = d \cdot 2^x$ because the value of $f(x)$ triples each time x is increased by one while the value of $g(x)$ doubles each time x is increased by one. Hence the graph of f is the one that intersects the y-axis at a lower value. The graph of f meets the y-axis at $f(0) = c \cdot 3^0 = c$ while the graph of g meets the y-axis at $g(0) = d \cdot 2^0 = d$. We conclude that $c < d$.

b. Along the graph of g each increase of one unit in the x value multiplies the output of g by 2. So an increase of two units in the x value multiplies the output of g by 4. Similarly, an increase of two units in the x value will multiply the value of f by $3^2 = 9$. So the ratio of my y-coordinate to your y-coordinate at our ending points is $\frac{9}{4}$.

Solution: Exponential Functions, Alternate Solution

a. Noting that $f(p) = g(p)$, we can say that $c \cdot 3^p = d \cdot 2^p$ so

$$\frac{c}{d} = \frac{2^p}{3^p} = \left(\frac{2}{3}\right)^p.$$

Since $p > 0$ it follows that $\left(\frac{2}{3}\right)^p < 1$. This means that $\frac{c}{d} < 1$ and $c < d$.

b. We have $\frac{f(p)}{g(p)} = 1$ since $f(p) = g(p)$. That means that $\frac{c \cdot 3^p}{d \cdot 2^p} = 1$. At the ending points, this ratio becomes

$$\frac{c \cdot 3^{p+2}}{d \cdot 2^{p+2}} = \frac{c \cdot 3^p}{d \cdot 2^p} \cdot \frac{3^2}{2^2} = 1 * \frac{9}{4} = \frac{9}{4}$$