Equal Differences over Equal Intervals 2

Task

a. Complete the table below. Is Δx constant? What constant is it? What do you notice about the 3rd column of the table?

x	$y = 3x - 4$	Δy
1	-1	$- - - -$
2	2	$2 - (-1) = 3$
3		
4		
5		

b. Complete the table below. Is Δx constant? What constant is it? What do you notice about the 3rd column of the table?

x	$y = 3x - 4$	Δy
1		$- - -$
3		
5		
7		
9		

c. Repeat the construction above the table for the linear equation $y = -2x + 1$. How do your observations in the 3rd column compare to those made for the previous table?

d. Let $y(x) = ax + b$. Let x_0 be any particular x-value. Show that if x_0 is increased by a constant Δx, the corresponding Δy is constant. What is this constant?

e. Is a) an example of the result of d)? Explain.

Commentary

This task assumes that students are familiar with the Δx and Δy notations. Students most likely developed this familiarity in their work with slope.

An important property of linear functions is that they grow by equal differences over equal intervals. In "Equal Differences over Equal Intervals 1", students prove this for equal intervals of length one unit, and note that in this case the equal differences have the same value as the slope. In "Equal Differences over Equal Intervals 2", students prove the property in general (for equal intervals of any length).

Instructors should use their judgment as to how many examples students should do on their own (for example, repeating part (c) with several different functions, possibly varying both the slope and the value of Δx) before attempting to tackle the more general algebraic scenario in part (d).

Solution

x	$y = 3x - 4$	Δy
1	-1	$- - - -$
2	2	$2 - (-1) = 3$
3	5	3
4	8	3
5	11	3

a. Δx, the change in successive x values, is constant and is 1. The numbers in the third column of the table are all 3s.

x	$y = 3x - 4$	Δy
1	-1	$- - -$
3	5	$5 - (-1) = 6$
5	11	6
7	17	6
9	23	6

b. Δx, the change in successive x values, is constant and is 2. The numbers in the third column of the table are all 6s.

c. Students should find a common difference of $\Delta y = -4$. In both this example and the previous, the common difference is precisely twice the slope.

d. At $x = x_0 + \Delta x$, $y = a(x_0 + \Delta a) + b$.

At $x = x_0$, $y = a(x_0) + b$. The difference between these two y-values is

$$\begin{aligned}[a(x_0 + \Delta x) + b] - [a \cdot x_0 + b] &= a \cdot x_0 + a \cdot \Delta x + b - a \cdot x_0 - b \\ &= a \cdot \Delta x.\end{aligned}$$

So the change in the y-value when x_0 is increased by Δx does not depend on x_0: it is always $a \cdot \Delta x$, the product of the constant Δx and the constant slope of the line described by the equation $y = ax + b$.

e. In a) the Δx was 1 and a, the slope of the line, was 3. The constant change in y-values was $3 = \Delta x \cdot a = 1 \cdot 3$. So a) is an example of the result in d).