Equal Differences over Equal Intervals 1

a. Complete the table below. In the third column, show your work as demonstrated. What do you notice about the 3rd column of the table?

x	$y = 2x + 5$	Δy
1	7	----
2	9	9 - 7 = 2
3		
4		
5		

b. Complete the table below, showing your work as above. What do you notice about the 3rd column of the table? What is the graphical interpretation of this?

x	$y = ax + b$	Δy
1	$a \cdot 1 + b$	---
2	$a \cdot 2 + b$	$a \cdot 2 + b - (a \cdot 1 + b) = a$
3		
4		
5		

c. Let $y = ax + b$. Let x_0 be any particular x-value. Show that if x_0 is increased by 1, the corresponding Δy is constant. What is this constant?

d. Does a) serve as an example of the result in c) ? Explain.

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Commentary

An important property of linear functions is that they grow by equal differences over equal intervals. In F.LE Equal Differences over Equal Intervals 1, students prove this for equal intervals of length one unit, and note that in this case the equal differences have the same value as the slope. In F.LE Equal Differences over Equal Intervals 2, students prove the property in general (for equal intervals of any length).

An alternative approach to this task would be to restructure it as a guided discovery lesson. Instead of asking students to examine first differences (as modeled by the three-column table provided) for one linear function and then jumping right to the general form, students could first examine such differences for a variety of linear functions with varying slopes (including negative slopes) and y-intercepts. From this they will be able to ascertain the relative effects of both the slope and the y-intercept and hopefully deduce a viable conjecture. At this point, challenge individuals or small cooperative groups of two or three, to construct an algebraic argument in support of their conjecture. It would also be instructive to ask students how their conclusions might change with increases in x values greater than or less than one, say increases of 2, 3, one-half, or k, segueing nicely into the follow-up problem mentioned above.

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Solution: Solution

a. The table of values of $y = 2x + 5$ and the corresponding Δy values are given in table below:

x	$y = 2x + 5$	Δy
1	7	---
2	9	$9 - 7 = 2$
3	11	$11 - 9 = 2$
4	13	$13 - 11 = 2$
5	15	$15 - 13 = 2$

The numbers in the third column of the table are all twos. In this case, $y = 2x + 5$ is the equation of a line with slope two; therefore each increase of one unit in the x-value produces an increase of two units in the y-value.

b. The table of values for $y = ax + b$ is given below:

x	$y = ax + b$	Δy
1	a+b	---
2	2a+b	(2a+b) - (a+b) = a
3	3a+b	(3a+b) - (2a+b) =a
4	4a+b	(4a+b) - (3a+b)=a
5	5a+b	(5a+b) - (4a+b)=a

The numbers in the third column of the table are all a's. In this case, $y = ax + b$ is the equation of a line with slope a; therefore each increase of one unit in the x-value produces an increase a in the y-value.

c. When $x = x_0 + 1$, $y = a(x_0 + 1) + b$.

When $x = x_0$, $y = ax_0 + b$.

The difference between these y-values is:

$$[a(x_0 + 1) + b] - [ax_0 + b] = (ax_0 + a + b) - (ax_0 + b) = a.$$

So the change in the value of y when x_0 is increased by one does not depend on the value of x_0: it is always a, the slope of the line described by the equation $y = ax + b$.

d. In a), as the x-values each increased by 1, the y-values increased by the 2, which was the coefficient of the x in $y = 2x + 5$. So a) is an example of the result in c), treating the cases where $x_0 = 1, 2, 3, 4$.

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