Braking Distance

Task
Suppose that a particular model of car has a braking distance that can be computed as follows: When the car is traveling at v miles per hour, its braking distance is given (in feet) by

$$d = 2.2v + \frac{v^2}{20}.$$

a. What is the braking distance, in feet, of a car of this model going 30 mph? 60 mph? 90 mph?

b. Suppose that a specific car of this model took 500 feet to brake. Use your computations in part (a) to make a prediction about how fast it was going when the brakes were applied.

c. Use a graph of the distance equation to determine more precisely how fast it was going when the brakes were applied, and check your answer using the quadratic formula.

Commentary
This task provides an exploration of a quadratic equation by descriptive, numerical, graphical, and algebraic techniques. Based on its real-world applicability, teachers could use the task as a way to introduce and motivate algebraic techniques like completing the square, en route to a derivation of the quadratic formula.

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Solution

a. At 30 mph, braking distance $= d = 2.2(30) + \frac{30^2}{20} = 111$ ft.

At 60 mph, braking distance $= d = 2.2(60) + \frac{60^2}{20} = 312$ ft.

At 90 mph, braking distance $= d = 2.2(90) + \frac{90^2}{20} = 603$ ft.

b. From our answers in part (a), we see that the speed for a braking distance of 500 ft is between 60 mph and 90 mph. Estimates within this range can vary, though based on the computations, it is reasonable to predict a velocity closer to 90 than to 60.

c. The figure shows a graph of $d = 2.2v + \frac{v^2}{20}$. We see that the speed corresponding to a braking distance of 500 ft is approximately 80 mph.

To find the speed more accurately, we use the quadratic formula to solve the quadratic equation:

$$500 = 2.2v + \frac{v^2}{20} \text{ which is } \frac{1}{20}v^2 + 2.2v - 500 = 0,$$

giving $v = 80.39$ and $v = -124.39$. On physical grounds we want $v > 0$, so $v = 80.39$, which is in good agreement with our graphical estimate of 80 mph.

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