Accurately weighing pennies I

In 1983 the composition of pennies in the United States was changed due, in part, to the rising cost of copper. Pennies minted after 1983 weigh 2.50 grams while the earlier copper pennies weigh 3.11 grams.

a. A roll of pennies contains 50 coins. If a particular roll of pennies weighs 138.42 grams, how many of the old heavier pennies and how many of the new lighter pennies does this roll contain?

b. Suppose you measure a roll of pennies on a scale which only reads the weight to the nearest gram. The scale says that the roll of pennies weighs 131 grams. What can you conclude about the pennies in this roll? What if the roll weighs 134 grams? Explain.

c. Some references list the weight of the newer pennies as 2.5 grams instead of 2.50 grams. If you only knew that the measurement of 2.5 grams was precise to the nearest tenth of a gram, what can you conclude about the pennies in the roll if the scale reads 138.4 grams?

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Commentary

This problem involves solving a system of algebraic equations from a context: depending how the problem is interpreted, there may be one equation or two. The main work in parts (a) and (b) is in setting up the equation(s) appropriately. Question (c) is more subtle and it requires thinking carefully about the accuracy available in a particular measurement (weight). The first two parts of this task could be used for instructional or assessment purposes while the third part should strictly be implemented for instructional purposes.

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Solution: 1 System of Equations

a. If we call x the number of older, heavier pennies in the roll and y the number of newer lighter pennies in the roll then we have the equation

$$x + y = 50$$

since there are a total of 50 pennies in the roll. Each of the older pennies weighs 3.11 grams while each of the newer pennies weighs 2.50 grams so the total weight of the pennies in the roll will be $3.11x + 2.50y$ giving us a second equation

$$3.11x + 2.50y = 138.42.$$

Multiplying the first equation by 5 gives $5x + 5y = 250$ while multiplying the second equation by 2 gives $6.22x + 5y = 276.84$. Subtracting the first from the second we find

$$1.22x = 26.84$$

or

$$x = \frac{26.84}{1.22} = 22.$$

So there are 22 of the older pennies and hence there must be 28 of the newer ones in order to make a full roll of 50.

b. One way to approach this problem would be to repeat the reasoning in part (a), using the equation $3.11x + 2.50y = 131$ instead of $3.11x + 2.50y = 138.42$. The solutions for x and y will not be whole numbers but the whole numbers of interest will be the nearest ones above and below the decimal values of x and y.

Another method would be informed guess and check. From part (a) we know that with 22 of the heavier pennies the weight is 138.42 grams which is too much. With no heavier pennies, the weight will be $50 \times 2.50 = 125$ grams. The over estimate is off by a little more than the underestimate so the answer should be a little less than half way between the two. If we try 11 of the heavier pennies we get a total weight of

$$11 \times 3.11 + 39 \times 2.50 = 131.71.$$

This is too much and the scale would read 132 grams. If we try 10 of the heavier pennies we find

$$10 \times 3.11 + 40 \times 2.50 = 131.1$$

so the scale would read 131 grams. We should also check what happens with 9 of the heavier pennies:

$$9 \times 3.11 + 41 \times 2.50 = 130.49$$

and the scale would read 130 grams. So if the scale reads 131 grams we can conclude that there are 10 of the heavier pennies and 40 of the lighter ones.

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Similar reasoning can be applied if the scale reads 134 grams. If there are 11 heavier pennies we saw that the weight is about 2.3 grams short of 134 grams. Each heavier penny increases the weight by a little over 0.6 grams so 14 heavier pennies should be about right for a weight of 134 grams:

$$14 \times 3.11 + 36 \times 2.50 = 133.54.$$

So this is one possibility. If we try 15 heavier pennies we find

$$15 \times 3.11 + 35 \times 2.50 = 134.15.$$

We can check that 16 pennies will read 135 grams and 13 pennies would read 133 grams. So if the scale reads 134 grams, then we can not conclude exactly how many heavier pennies are in the roll: there could be either 14 or 15.

c. We know from part (a) that one possibility is that there are 22 of the heavier pennies and 28 lighter ones. This is still possible since each of the lighter pennies could, for example, weigh 2.50 grams, giving a total of 70 grams while the 22 heavier ones would contribute 68.42 grams. But if the lighter pennies weigh a little more than 2.5 grams then there could be more of them and if they weigh a little less than 2.5 grams there could be fewer.

The error in measurement for each lighter penny is $.05$ grams since the scale measures to the nearest tenth of a gram. So with 28 of the lighter pennies their possible weight could be off by $28 \times 0.05 = 1.4$ grams. Since the difference in weight between the heavier and lighter pennies is 0.61 grams, this is enough to compensate for 2 of the heavy pennies, more or less, but not enough to compensate for 3. So if the lighter pennies are known to weigh 2.5 grams, to the nearest tenth of a gram, then a roll of pennies of weight 138.4 grams could contain $26, 27, 28, 29,$ or 30 of the lighter pennies. If there were 26 of the lighter pennies, they would almost all have to be close to 2.45 grams in weight and similarly if there were 30 of the lighter pennies they would all have to be close to 2.55 grams in weight.

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Solution: 2: Single Equation

a. Each of the heavier pennies weighs $2.50 + 0.61$ grams. So if there are x of the heavier pennies in the roll then the total weight of pennies in the roll is

$$50 \times 2.50 + 0.61x.$$

This is the same as $125 + 0.61x$. So if the total weight of the roll is 138.42 grams then we have

$$125 + 0.61x = 138.42.$$

Solving for x gives

$$x = \frac{13.42}{0.61} = 22$$

so there are 22 of the heavier pennies and 28 of the lighter ones.

b. As in part (a), the total weight of the pennies in the roll is

$$125 + 0.61x$$

grams where x is the total number of heavier pennies in the roll. So if the scale reads 131 grams, to the nearest gram, this means that $0.61x$ must be 6, to the nearest gram. This means that $x = 10$ so there are ten of the heavier pennies. If the scale reads 134 grams then we have, similarly, that $0.61x$ must be, rounded to the nearest unit, 9. This is possible if $x = 14$ or if $x = 15$. Thus in this case there either 14 or 15 of the heavier pennies.

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