

Same solutions?

Task

Which of the following equations have the same solution? Give reasons for your answer that do not depend on solving the equations.

a. $x + 3 = 5x - 4$

b. $x - 3 = 5x + 4$

c. $2x + 8 = 5x - 3$

d. $10x + 6 = 2x - 8$

e. $10x - 8 = 2x + 6$

f. $0.3 + \frac{x}{10} = \frac{1}{2}x - 0.4$



Commentary

The purpose of this task is to provide an opportunity for students to reason about equivalence of equations. The instruction to give reasons that do not depend on solving the equation is intended to focus attention on the transformation of equations as a deductive step.

Note that although it is possible to show that two equations are equivalent without solving them, it is more difficult to give reasons why they are not equivalent, even though they do not appear to be. Thus, in the end, confirmation of the solution is achieved by solving the equations.



Solution

Equation (e) is Equation (a) multiplied by 2, and with the left side written on the right.

Equation (f) is Equation (a) divided by 10 with the two terms on the left written in the opposite order. So Equations (a), (e), and (f) all have the same solutions.

Equation (b) has the signs of the constants changed from Equation (a), so probably does not have the same solution.

Equation (c) has two of the terms in Equation (a), the x and -4 , multiplied by 2 while the other two are not. It does not have the same solutions as equations (a) or (b).

Equation (d) is Equation (b) multiplied by 2, with the constant terms moved to the opposite side of the equation. Equations (b) and (d) have the same solutions.

Confirmation: The solutions to the equations are:

Equation	a	b	c	d	e	f
Solution	$7/4$	$-7/4$	$11/3$	$-7/4$	$7/4$	$7/4$

