Carbon 14 dating in practice II

Task

In order to use Carbon 14 for dating, scientists measure the *ratio* of Carbon 14 to Carbon 12 in the artifact or remains to be dated. When an organism dies, it ceases to absorb Carbon 14 from the atmosphere and the Carbon14 within the organism decays exponentially, becoming Nitrogen 14, with a half-life of approximately 5730 years. Carbon 12, however, is stable and so does not decay over time.

Scientists estimate that the ratio of Carbon 14 to Carbon 12 today is approximately 1 to 1,000,000,000,000.

a. Assuming that this ratio has remained constant over time, write an equation for a function which models the ratio of Carbon 14 to Carbon 12 in a preserved plant t years after plant has died.

b. In a particular preserved plant, the ratio of Carbon 14 to Carbon 12 is estimated to be about 1 to 13,000,000,000. What can you conclude about when plant lived? Explain.

c. Dinosaurs are estimated to have lived from about 230,000,000 years ago until about 65,000,000 years ago. Using this information and the given half-life of Carbon 14, explain why this method of dating is not used for dinosaur remains.

Commentary

This problem introduces the method used by scientists to date certain organic material. It is based not on the amount of the Carbon 14 isotope remaining in the sample but rather on the ratio of Carbon 14 to Carbon 12. This ratio decreases, hypothetically, at a constant exponential rate as soon as the organic material has ceased to absorb Carbon 14, that is, as soon as it dies.

Carbon 14 dating is a fascinating topic and much information can be found on Wikipedia.

Many factors limit the accuracy of using Carbon 14 for dating including

a. the hypothesis that levels of Carbon 14 in the environment have been relatively constant. These levels can be influenced by climate, by natural processes such as volcanoes, and in recent times, by human activity.

b. the accuracy of measurement for the amount of Carbon 14 in a given sample. This is a serious issue because the current ratio of 1 to 1,000,000,000 means that extremely precise measurements will be needed to determine how much Carbon 14 is in a specimen.

c. the method used to estimate the amount of Carbon 14 in a given sample. If this is done by measuring the current decaying Carbon 14 then it is not statistically reliable with very small samples. More recent technology actually allows scientists to measure the remaining Carbon 14 much more accurately.

This problem is intended for instructional purposes only. It provides an interesting and important example of mathematical modeling with an exponential function. If the teacher has the time and inclination, it also reveals many of the inherent difficulties with mathematical modeling, some of which are mentioned in the previous paragraph as regards this particular example.

Solution

a. Suppose d represents the amount of Carbon 14 in the plant at its time of death while b represents the total amount of Carbon 12. As time progresses, does not change while d decreases exponentially. Hence the ratio $(d: b)$ also decreases exponentially. At the time of the plants' death, we have, assuming the same ratio of Carbon 14 to Carbon 12 as today, that

$$\frac{d}{b} = \frac{1}{1{,}000{,}000{,}000{,}000}.$$

Let f be the function which assigns to t, the number of years since the plant's death, the ratio of Carbon 14 remaining in the preserved plant to Carbon 12. So

$$f(0) = \frac{d}{b} = \frac{1}{1{,}000{,}000{,}000}.$$

Since the ratio of Carbon 14 to Carbon 12 is cut in half every 5730 years, this means that the equation

$$f(t) = \left(\frac{1}{1{,}000{,}000{,}000}\right)\left(\frac{1}{2}\right)^{\frac{t}{5730}}.$$

models the ratio of Carbon 14 to Carbon 12 if the input t is in years and the output $f(t)$ is measured in micrograms. Indeed, f satisfies $f(0) = \frac{1}{1{,}000{,}000{,}000}$ and the values of f for positive t decrease exponentially, being cut in half every 5730 years.

b. If the ratio of Carbon 12 to Carbon 14 is 1 to 13,000,000,000,000this means that

$$f(t) = \frac{1}{13{,}000{,}000{,}000{,}000}$$

where f was the function we computed in part (a). So we have

$$\left(\frac{1}{1{,}000{,}000{,}000{,}000}\right)\left(\frac{1}{2}\right)^{\frac{t}{5730}} = \frac{1}{13{,}000{,}000{,}000{,}000}.$$

This means that

$$\left(\frac{1}{2}\right)^{\frac{t}{5730}} = \frac{10}{13}.$$

Taking the logarithm, with base 2, on both sides we find

$$\frac{t}{5730}\log_2\left(\frac{1}{2}\right) = \log_2\left(\frac{10}{13}\right).$$

Solving for t using a calculator we find that it has been about 21,200 years since the plant died.

c. If a dinosaur died 65,000,000 years ago, this represents over 11,000 half lives for Carbon 14. Since $2^{10} = 1024$ is a little more than a thousand, this means that

$$2^{10,000} = (2^{10})^{1000} > 1000^{1000}.$$

So after being halved 11,000 times the amount of Carbon 14 remaining in the fossil would be zero or negligible since any measurable remains of Carbon 14 would imply that the dinosaur had more than 1000^{1000} times that measurable amount of Carbon 14 when it lived and this is not possible. For this reason, together with the limitations of technology to measure very small amounts of Carbon 14, Carbon 14 is only used to date artifacts up to about 60,000 years old.