Basketball Rebounds

Task

According to Wikipedia, the International Basketball Federation (FIBA) requires that a basketball bounce to a height of 1300 mm when dropped from a height of 1800 mm.

a. Suppose you drop a basketball and the ratio of each rebound height to the previous rebound height is 1300:1800. Let h be the function that assigns to n the rebound height of the ball (in mm) on the nth bounce. Complete the chart below, rounding to the nearest mm.

n	h(n)
0	1800
1	
2	
3	

b. Write an expression for h(n).

c. Solve an equation to determine on which bounce the basketball will first have a height of less than 100 mm.

Solution

a. On the first rebound, the ball must rise to a height of 1300 mm according to the IBF regulations.

To determine the second rebound height, 1300 mm must be multiplied by the required fraction $\frac{1300}{1800} = \frac{13}{18}$:

$$1300\left(\frac{13}{18}\right) = \frac{16{,}900}{18} \approx 939 \text{ mm.}$$

For each succeeding rebound the previous rebound height is multiplied by $\frac{13}{18}$;

n	h(n)
0	1800
1	1300
2	$1300\left(\frac{13}{18}\right) \approx 939$
3	$938.9\left(\frac{13}{18}\right) \approx 678$

n	h(n)
0	1800
1	$1300 = 1800\left(\frac{1300}{1800}\right)$
2	$938.9 \approx 1300\left(\frac{13}{18}\right) = 1300\left(\frac{13}{18}\right)\left(\frac{13}{18}\right) = 1800\left(\frac{13}{18}\right)^2$
3	$678.1 \approx 938.9\left(\frac{13}{18}\right) \approx 1800\left(\frac{13}{18}\right)^2 \cdot \left(\frac{13}{18}\right) = 1800\left(\frac{13}{18}\right)^3$
n	$1800\left(\frac{13}{18}\right)^n$

b. Generalizing from the table, we have $h(n) = 1800\left(\frac{13}{18}\right)^n$: this makes sense because by the n^{th} rebound, we've multiplied by $\frac{13}{18}$, n times.

c. The rebound on which the height will be approximately 100 mm can be determined by solving for n given $h(n) = 100$:

$$100 = 1800\left(\frac{13}{18}\right)^n$$

or

$$\frac{1}{18} = \left(\frac{13}{18}\right)^n$$

Logarithms may be used to solve such an equation:

$$\ln\left(\frac{1}{18}\right) = \ln\left(\frac{13}{18}\right)^n$$
$$\ln\left(\frac{1}{18}\right) = n \cdot \ln\left(\frac{13}{18}\right)$$

rounding yields the equation

$$-2.890 \approx n \cdot (-0.325)$$

which tells us

$$n \approx 8.892$$

So, the first time the rebound will not be at least 100 mm with be on the 9^{th} rebound.