Writing a Mixed Number as an Equivalent Fraction

Task

Ben wrote a mixed number as the fraction $\frac{1}{3}$. Here is his work:

$$
\begin{aligned}
7\frac{1}{3} &= \frac{7}{1} + \frac{1}{3} && \text{(Step 1)} \\
&= \frac{(7 \times 3) + 1}{3} && \text{(Step 2)} \\
&= \frac{21 + 1}{3} && \text{(Step 3)} \\
&= \frac{22}{3} && \text{(Step 4)}
\end{aligned}
$$

Explain what Ben did in each step.

Commentary

The purpose of this task is to help students understand and articulate the reasons for the steps in the usual algorithm for converting a mixed number into an equivalent fraction, that is

$$A\frac{B}{C} \qquad \frac{A \quad C \ +B}{C}$$

Step two shows that the algorithm is merely a shortcut for finding a common denominator between two fractions. This concept is an important precursor to adding mixed numbers and fractions with like denominators and as such, step two should be a point of emphasis.

This task is appropriate for either instruction or formative assessment. If the goal is to get students to explain the steps using pictures, then more guidance might be necessary.

A natural extension or complement to this task would be to perform and justify the reverse of these steps, that is, going from a fraction greater than to a mixed number.

Solutions

Solution: Solution 1

Recall that $\frac{1}{3}$ is shorthand for $\frac{1}{3}$, and that $\frac{7}{1}$.

So in Step , Ben is writing $\frac{1}{3}$ as $\frac{1}{3}$ and then writing as $\frac{7}{1}$.

In Step , Ben found an equivalent fraction for $\frac{7}{1}$ so that it would have the same denominator as $\frac{1}{3}$:

and then he added them together

In Step 3, Ben found the product .

In Step 4, he added and in the numerator.