

# Double Plus One

## Task

- a. The table below shows a list of numbers. For every number listed in the table, multiply it by 2 and add 1. Record the result on the right.

Number	Double the Number Plus One
0	
1	
2	
3	
4	
5	
10	
23	
57	
100	
309	

- b. What do you notice about the numbers you entered into the table?
- c. Sherri noticed that all the numbers she entered are odd.
- Does an even number multiplied by 2 result in an even or odd number? Why do you think this is?
  - Does an odd number multiplied by 2 result in an even or odd number? Why do you think this is?
  - Does an even number plus 1 result in an even or odd number? Why do you think this is?
  - Does an odd number plus 1 result in an even or odd number? Why do you think this is?
  - Explain why the numbers you entered in the table are all odd.

## Commentary

This task is meant to be used in an instructional setting. Part (b) of this task is intentionally left open-ended to encourage students to develop the habit of looking for patterns that might hint at some underlying structure.

This kind of work is also related to making conjectures and determining whether those conjectures are true or not. Some of the things that students might notice about the numbers in the table don't really go anywhere; in fact, that is the nature of making conjectures: they don't always turn out to be true.

Part (c) is meant to engage students in what should naturally follow after noticing a pattern, namely, investigating whether it always holds and if so, explaining why.

A variant on this task would remove the scaffolding from part (c). In either case, having small objects, like chips or coins, might help students develop their explanations.

Submitted by Sherri Martinie and Melisa Hancock of Kansas State University, Manhattan, KS to the first Illustrative Mathematics Task writing contest.



## Solution

- a. The table below shows a list of numbers. For every number listed in the table, multiply it by 2 and add 1. Record the result on the right.

Number	Double the Number Plus One
0	1
1	3
2	5
3	7
4	9
5	11
10	21
23	47
57	115
100	201
309	619
$n$	$2n + 1$

- b. There are several patterns that students might see. For example, they might notice that the result increases by two when the number increases by one. Hopefully, some students will notice that all the numbers they entered are odd.
- c. Sherri noticed that all the numbers she entered are odd.
- i. An even number multiplied by 2 result in an even number, because multiplying by 2 will double the number of objects. As a result, if your objects are paired off, every object will have a partner.

Example: We can represent six multiplied by 2 as 2 rows with six circles in each row:



As you can see, every circle has a partner.

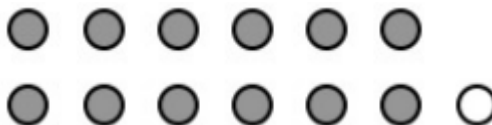
- ii. An odd number multiplied by 2 result in an even number, because multiplying by 2 will double the number of objects. As a result, if your objects are paired off, every object will have a partner.

Example: We can represent seven multiplied by 2 as 2 rows with seven circles in each row:



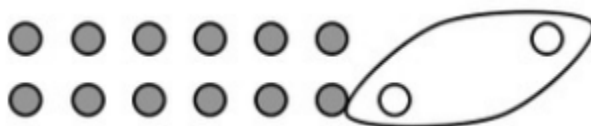
- iii. An even number plus 1 is odd, because an even number of objects can be paired off but adding only one more object will mean that the last object added will not have a partner: there will be one left over when you try to pair them off.

Example: If we represent 12 (an even number) as two rows of six circles, and add one more circle, we can see it must be odd:



- iv. An odd number plus 1 is even, because when you have an odd number of objects and try to pair them off, one will always be left over. If you add one more object, then the lone object gains a partner, and all the objects will be in pairs.

Example: If we represent 13 (an odd number) as two rows of six circles and one lonely circle, and then add one more circle, we can see the “extra” circle now has a partner:



- v. The values in the right column are all odd because multiplying an even or an odd number by 2 always results in an even number. An even number plus 1 will always result in an odd number, as shown above.