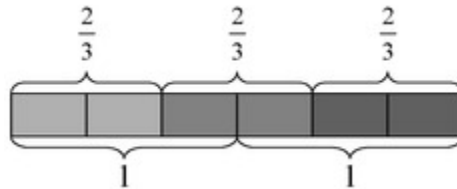


Connor and Makayla Discuss Multiplication

Task

Makayla said, "I can represent $3 \times \frac{2}{3}$ with 3 rectangles each of length $\frac{2}{3}$."



Connor said, "I know that $\frac{2}{3} \times 3$ can be thought of as $\frac{2}{3}$ of 3. Is 3 copies of $\frac{2}{3}$ the same as $\frac{2}{3}$ of 3?"

- Draw a diagram to represent $\frac{2}{3}$ of 3.
- Explain why your picture and Makayla's picture together show that $3 \times \frac{2}{3} = \frac{2}{3} \times 3$.
- What property of multiplication do these pictures illustrate?

Commentary

The purpose of this task is to

- a. Have students think about the meaning of multiplying a number by a fraction, and
- b. Use this burgeoning understanding of fraction multiplication to make sense of the commutative property of multiplication in the case of fractions.

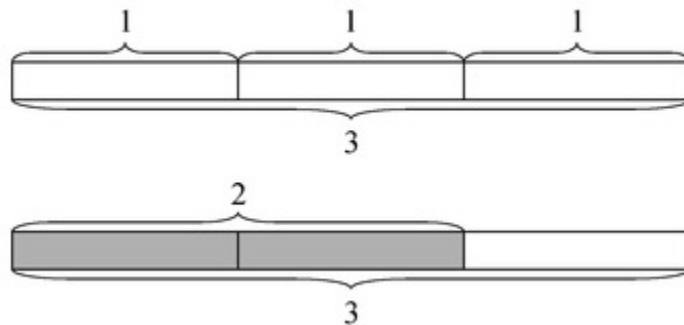
In principle, students have already had a chance to see why multiplication of whole numbers is commutative by looking at arrays or rectangles in third grade. After seeing the kinds of pictures used in this task and its solution, students should also look at rectangles with fractional side lengths to help reinforce their understanding of the commutative property of multiplication.

Students should have some prior experience with interpreting $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts before they are ready to tackle parts (b) and (c) of the task.



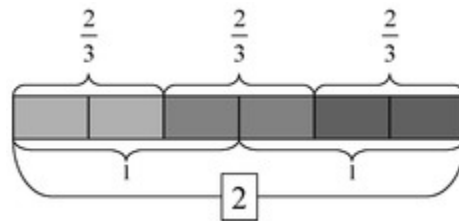
Solution

- a. A rectangle of length 3 can be built from three rectangles of length 1:



Thus, $\frac{2}{3}$ of the large rectangle is 2 of the smaller rectangles of length 1.

- b. We can see in Makayla's picture that there are 3 groups of 2 thirds, so there are 6 thirds altogether. Since 3 thirds is 1 whole, 6 thirds is $6 \div 3 = 2$ wholes. So three rectangles each of length $\frac{2}{3}$ has a total length of 2.



In the picture for part (a), we can see that $\frac{2}{3}$ of 3 is also 2. So this shows that 3 groups of $\frac{2}{3}$ (an interpretation of $3 \times \frac{2}{3}$) is equal to $\frac{2}{3}$ of 3 (an interpretation of $\frac{2}{3} \times 3$), or

$$3 \times \frac{2}{3} = \frac{2}{3} \times 3$$

- c. These pictures illustrate the commutative property of multiplication.