The High School Gym

Task

The school assembly is being held over the lunch hour in the school gym. All the teachers and students are there by noon and the assembly begins. About 45 minutes after the assembly begins, the temperature within the gym remains a steady 77 degrees Fahrenheit for a few minutes. As the students leave after the assembly ends at the end of the hour, the gym begins to slowly cool down.

Let T denote the temperature of the gym in degrees Fahrenheit and M denote the time, in minutes, since noon.

a. Is M a function of T? Explain why or why not.

b. Explain why T is a function of M, and consider the function $T = g(M)$. Interpret the meaning of $g(0)$ in the context of the problem.

c. Becky says: "The temperature increased 5 degrees in the first half hour after the assembly began." Which of the following equations best represents this statement? Explain your choice.

   i. $g(30) = 5$

   ii. $\frac{g(30)-g(0)}{30} = 5$

   iii. $g(30) - g(0) = 5$

   iv. $T = g(30) - 5$

d. Which of these choices below represents the most reasonable value for the quantity $\frac{g(75)-g(60)}{15}$? Explain your choice:

   i. 4

   ii. 0.3

   iii. 0

   iv. -0.2

   v. -5

Solution

a. No, M is not a function of T. The problem states that the temperature, T, remains a steady 77 degrees Fahrenheit for a few minutes, M. Thus, we know that there is more than one value of M, or more than one minute, associated with the temperature $T = 77$. This means that if M were a function of T there would be more than one output M associated with the single input value of $T = 77$. By definition of a function each input must be associated with exactly one output. Therefore, M cannot be a function of T because there would be more than one output associated

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with the input $T = 77$.

b. In contrast the previous part, it is the case that for each number of minutes M after noon, there is one and only one temperature T in the room, so T is a function of M. The quantity $g(0)$ is the value of T when $M = 0$. From the context given in the problem we know that M denotes the time in minutes since noon. Thus, $M = 0$ is exactly noon, so $g(0)$ is the temperature in the high school gym in degrees Fahrenheit at exactly noon.

c. Let's look at the meaning of each equation:

   i. $g(30) = 5$ says the temperature at 12:30 p.m. is 5 degrees Fahrenheit, which is not what Becky said.

   ii. $\frac{g(30)-g(0)}{30} = 5$ says that the average rate of change of the temperature of the gym between noon and 12:30 p.m. is 5 degrees per minute, which is not what Becky said.

   iii. $g(0) - g(30) = 5$ says that the difference in the temperature of the gym at 12:30 and the temperature of the gym at noon is 5 degrees, which is the same thing as saying the temperature increased by 5 degrees Fahrenheit.

   iv. $T = g(30) - 5$ says that the temperature equals the temperature at 12:30 minus five degrees, which is not what Becky said.

d. As in the previous part, we recognize the quantity $g(75) - g(60)$ as the change in the temperature in the room in the 15 minutes following the students' departure from the gym. We are told that the temperature decreases as the students begin to leave, so we know this quantity must be negative.

   Next, we note that the quantity in question, namely $\frac{g(75)-g(60)}{15}$, is precisely the average rate of change of the temperature over these first fifteen minutes, and so is also negative. This eliminates the first three choices for the value of this quantity immediately.

   Finally, we note that the last option corresponds to an average decrease of 5 degrees per minute, which would combine for a 75-degree drop in this 15-minutes span. Since this would leave the gym in a frozen state, we conclude that this was an implausibly fast rate of temperature decrease. This leaves only the value -0.2, corresponding to a significantly more plausible temperature drop of 3 degrees over these 15 minutes.

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