Pizza Place Promotion

Alignment: MAFS.912.F-IF.2

Task

In order to gain popularity among students, a new pizza place near school plans to offer a special promotion. The cost of a large pizza (in dollars) at the pizza place as a function of time (measured in days since February 10th) may be described as

$$C(t) = \begin{cases} 9, & 0 \leq t \leq 3 \\ 9 + t, & 3 < t \leq 8 \\ 20, & 8 < t < 28 \end{cases}$$

(Assume t only takes whole number values.)

a. If you want to give their pizza a try, on what date(s) should you buy a large pizza in order to get the best price?

b. How much will a large pizza cost on Feb. 18th?

c. On what date, if any, will a large pizza cost 13 dollars?

d. Write an expression that describes the sentence "The cost of a large pizza is at least A dollars B days into the promotion," using function notation and mathematical symbols only.

e. Calculate $C(9) - C(8)$ and interpret its meaning in the context of the problem.

f. On average, the cost of a large pizza goes up about 85 cents per day during the first two weeks of the promotion period. Which of the following equations best describes this statement?

- $\frac{C(13)+C(0)}{2} = 0.85$
- $\frac{C(13)-C(0)}{13} = 0.85$
- $\frac{C(13)}{13} = 0.85$
- $\frac{C(Feb.23)-C(Feb.10)}{13} = 0.85$

Solution

a. Based on the function above, the lowest price that the promotion offers for a large pizza is 9 dollars. This is the cost of the pizza when $t = 0$, $t = 1$, $t = 2$, and $t = 3$. We know that t denotes the number of days since February 10^{th}. Thus, $t = 0$ corresponds to February 10^{th}, $t = 1$ corresponds to February 11^{th}, $t = 2$ corresponds to February 12^{th}, and $t = 3$ corresponds to February 13^{th}. Therefore, the best days to give the new pizza place a try in order to get the best price are February 10, February 11, February 12 and February 13.

b. February 18^{th} is eight days after February 10^{th}, corresponding to $t = 8$. Based on the function above, the cost of a large pizza in dollars is given by $C(t) = 9 + t$ when $3 < t \leq 8$. Then, when $t = 8$, $C(t) = 9 + 8 = 17$ dollars. Thus, the cost of a large pizza on February 18th is 17 dollars.

c. We know that $13 \neq 9$ which implies that a large pizza cannot cost 13 dollars when $0 \leq t \leq 3$ because for these values of t, $C(t) = 9$ based on the function above. Similarly, we know that $13 \neq 20$, which implies that a large pizza cannot cost 13 dollars when $8 < t < 28$ because $C(t) = 20$ for these values. Thus, we know that the only time a large pizza could cost 13 dollars is when t is in the interval $3 < t \leq 8$, for which $C(t) = 9 + t$. In order to find out which date a large pizza will cost 13 dollars we must plug 13 into this equation as our cost and solve for t:

$$\begin{aligned} 13 &= 9 + t \\ t &= 13 - 9 \\ t &= 4 \end{aligned}$$

This means that a large pizza costs 13 dollars when $t = 4$. We know that t denotes the days since February 10^{th} so $t = 4$ corresponds to February 14^{th}. Thus, a large pizza will cost 13 dollars on February 14^{th}.

d. The statement that a pizza is at least A dollars B days into the promotion means that the cost of a large pizza B days into the promotion, denoted $C(B)$, is greater than or equal to A dollars. Thus, an expression that describes this sentence using function notation and mathematical symbols is simply

$$C(B) \geq A$$

e. $C(9) = 20$ and $C(8) = 9 + 8 = 17$ based on the equation above. Thus, $C(9) - C(8) = 20 - 17 = 3$. $C(9) = 20$ corresponds to the cost of a large pizza 9 days after February 10^{th}, or February 19^{th}. $C(8) = 17$corresponds to the cost of a large pizza 8 days after February 10^{th}, or February 18^{th}. Thus, the meaning of $C(9) - C(8) = 3$ in the context of the problem is that on February 19^{th}, a large pizza will be 3 dollars more expensive than a large pizza on February 18^{th}.

f. The first two weeks of the promotion take place from February 10th, when $t = 0$, to February 23rd, when $t = 13$. The average rate of change is given by:

$$\frac{C(13) - C(0)}{13 - 0} = \frac{20 - 9}{13} \approx 0.85$$

The expression above shows that the cost of a large pizza goes up about 0.85 dollars, or 85 cents, per day during the first two weeks of the promotion period. Therefore, the expression that best describes this statement is:

$$\frac{C(13) - C(0)}{13}$$