

Average Cost

Task

John makes DVDs of his friend's shows. He has realized that, because of his fixed costs, his average cost per DVD depends on the number of DVDs he produces. The cost of producing x DVDs is given by

$$C(x) = 2500 + 1.25x.$$

- John wants to figure out how much to charge his friend for the DVDs. He's not trying to make any money on the venture, but he wants to cover his costs. Suppose John made 100 DVDs. What is the cost of producing this many DVDs? How much is this *per DVD*?
- John is hoping to make many more than 100 DVDs for his friends. Complete the table showing his costs at different levels of production.

# of DVDs	0	10	100	1,000	10,000	100,000	1,000,000
Total Cost							
Cost per DVD							

- Explain why the average cost per DVD levels off.
- Find an equation for the average cost per DVD of producing x DVDs.
- Find the domain of the average cost function.
- Using the data points from your table above, sketch the graph of the average cost function. How does the graph reflect that the average cost levels off?

Solution

- John's total cost for producing 100 DVDs would be $C(100) = \$2625$ (from the equation given), meaning that he would be spending an average of $2625/100 = \$26.25$ per DVD.
- John is hoping to make many more than 100 DVDs for his friend. Complete the table showing his costs at different levels of production.

# of DVDs	0	10	100	1,000	10,000	100,000	1,000,000
Total Cost	0	2512.5	2625	3750	15000	127500	1252500
Cost per DVD	x	251.25	26.25	3.75	1.5	1.275	1.2525

- The reason the average cost is changing is because of the fixed cost of \$2500. As more DVDs are produced, this fixed cost is shared by so many DVDs that it barely adds anything to the cost of each DVD. The average cost levels off at 1.25, which is what each additional DVD adds to the cost function.
- We are given the total cost function, $C(x) = 2500 + 1.25x$, for producing x DVDs. To find the average cost function, we just divide the total cost by the number, x , of DVDs produced to arrive at the function $\frac{2500+1.25x}{x}$.
- The domain for the average cost function is the set of positive integers. It obviously makes no sense to consider producing a negative or fractional number of DVDs, and you cannot compute an *average* cost if there are no DVDs produced, so the domain cannot include 0 either.
- The graph of the average cost function $\frac{2500+1.25x}{x}$ is shown below (for $x > 0$). As you follow the graph to the right (i.e., as the number of DVDs produced increases), the graph gets closer and closer to the dotted line, which is a *horizontal asymptote* of the graph. Note that the horizontal asymptote is necessarily the line $y = 1.25$, since as is explained in the solution for part (c), this is the long-term average cost of producing each additional DVD.

