US Households

Task

The table below shows the number of households in the U.S. in the years $1998 - 2004$ [data source: United States Census].

year	1998	1999	2000	2001	2002	2003	2004
households (in thousands)	97,107	98,990	99,627	101,018	102,528	103,874	104,705

a. Find a linear function, h, which models the number of households in the U.S. (in thousands) as a function of the year, t.

b. Write an expression for h^{-1}.

c. Find $h^{-1}(111{,}000)$ and interpret your answer in terms of the number of households.

Commentary

The purpose of this task is to construct and use inverse functions to model a real-life context. Students choose a linear function to model the given data, and then use the inverse function to interpolate a data point. We caution that given the open-ended nature of the modelling in this problem (where by choosing different data points they end up with different linear functions), the task is not intended to illustrate any of the statistics content concerning finding lines of best fit. In particular, we caution that reasoning with inverses of best-fit lines involves some subtlety: The least squares regression equation to predict x from y is not necessarily the inverse of the regression equation used to predict y from x. This task instead focuses on the interplay between modeling, predictions, and inverse functions, rather than the statistical process of finding a line of best fit.

Note: While students were not required to use f^{-1} notation for a, in b, c, and, d, students begin to use this notation. A common student error is to mistake f^{-1} for $\frac{1}{f}$, and students can be asked to notice that in this problem the expression for $h^{-1}(x)$ is not equivalent to the expression for $\frac{1}{h(x)}$.

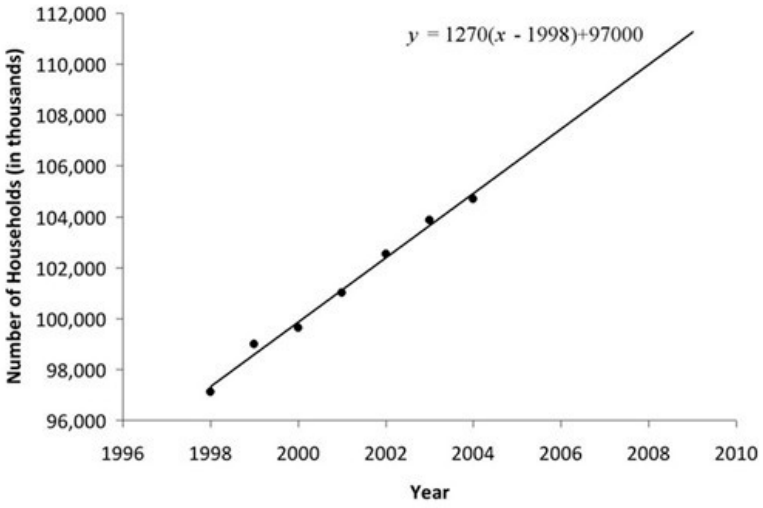
Adapted from a problem by Hilton Russell.

Solution

a. The average rate of change in the number of households from 1998 to 2004 is

$$\frac{104{,}705-97{,}107}{2004-1998} \approx 1270 \text{ thousand/year.}$$

We choose this rate as the annual rate of change in our model, and we approximate the number of households in 1998 by 97,000. This leads to the the linear function $h(t) = 1270(t - 1998) + 97{,}000$.



b. The original function first subtracted 1998 from its input, then multiplied by 1270, and finally added 97,000. Thus, the inverse function would subtract 97,000 from its input, divide by 1270, and then add 1998. Thus,

$$h^{-1}(x) = \frac{(x - 97{,}000)}{1270} + 1998$$

c. Using the formula for the inverse function from b), we get $h^{-1}(111{,}000) \approx 2009$. This is the year the number of U.S. households reached 111,000 thousand. Of course, this is extrapolating well beyond the data, so this prediction must be viewed with caution.