Summer Intern

Alignment: MAFS.912.F-BF.1.1

You have been hired for a summer internship at a marine life aquarium. Part of your job is diluting brine for the saltwater fish tanks. The brine is composed of water and sea salt, and the salt concentration is 15.8% by mass, meaning that in any amount of brine the mass of salt is 15.8% of the total mass.

a. The supervisor asks you to add fresh water to one liter of the brine using a half-liter measuring cup. Let $S(x)$ be the salt concentration of the resulting mixture when you add x half-liters of salt. Write an expression for $S(x)$. [Assume that one liter of water has mass 1 kg.]
b. Describe how the graph of S is related to the graph of $y = 1/x$.
c. Sketch the graph of S.
d. How much fresh water should you add to get a mixture which is 4% sea salt, approximately the salt concentration of the ocean?

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Solution: Summer Intern

a. Because the brine is 15.8% sea salt, the initial salt concentration is

$$\frac{0.158 \text{ kilograms of salt}}{1 \text{ kilogram of brine}}.$$

Adding x half-liters of water is the same as adding x half-kilograms of mass. This increases the total mass of the mixture by the x half-kilograms, to $0.5x+1$, while leaving the mass of salt unchanged. So an expression for $S(x)$ is

$$\frac{0.158}{0.5x+1}$$

b. We write the expression for $S(x)$ in a form that shows its relation with $1/x$:

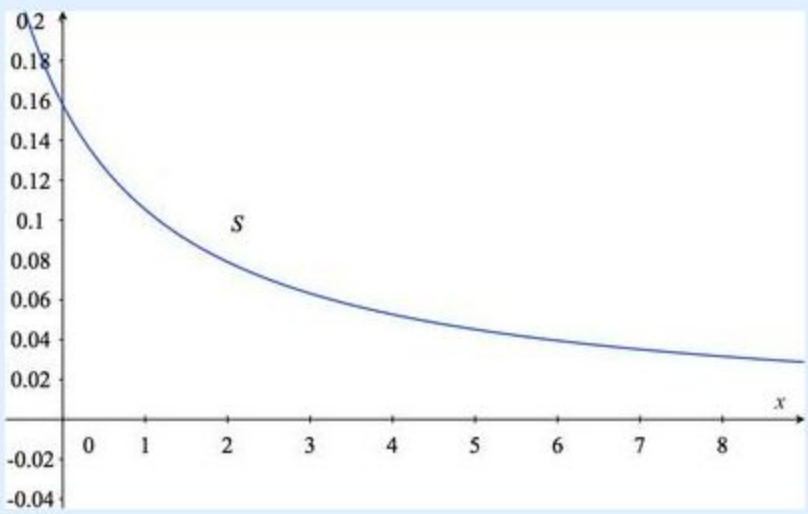
$$\frac{0.158}{0.5x+1} = \frac{0.158}{0.5(x+2)} = \frac{0.316}{x+2} = 0.316\,\frac{1}{x+2}.$$

Thus $S(x)$ is obtained from $1/x$ by first replacing x with $x+2$ and then multiplying the whole by 0.316. So the graph of S is the graph of

$$y = \frac{1}{x}$$

horizontally translated by 2 units to the left, and vertically dilated by 0.316 units.

c.



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d. Since a solution which is 4% sea salt has a salt concentration of 0.04, we must find x satisfying $S(x) = 0.04$. So, we solve the equation $\frac{0.316}{x+2} = 0.04$ for x:

$$\begin{aligned}
\frac{0.316}{x+2} &= 0.04 \\
0.316 &= 0.04(x+2) \\
\frac{0.316}{0.04} &= x+2 \\
7.9 &= x+2 \\
5.9 &= x.
\end{aligned}$$

So to each liter of 15.8% brine we would have to add about 6 half-liters of fresh water (3 liters) in order to get a 4% sea salt solution.

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Solution: A tabular approach to part (d)

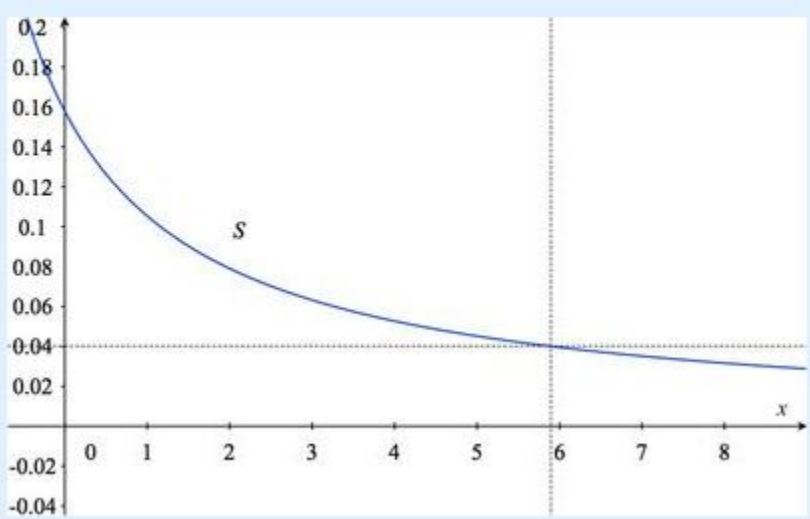
Using a calculator or spreadsheet we construct a table of values of $S(x)$ from the expression for $S(x)$ found in (b):

Number of half-liters	Salt concentration
0	0.158
1	0.153
2	0.079
3	0.0632
4	0.0527
5	0.045
6	0.0395
7	0.035

The table shows that the salt concentration is very close to 4% when 6 half-liters (3 liters) of fresh water are added to the brine.

Solution: A graphical approach to part (d)

The solution to part (d) can be estimated from the graph found in (c) by drawing a horizontal line with vertical coordinate 0.04 and seeing what the value of x is where it intersects the graph of S:



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