Rainfall

Alignment: MAFS.912.F-BF.2.4.c

The table below shows $R = f(t)$, the total amount of rain, in centimeters (cm), during a steady rainfall as a function of time, t, in minutes, since the rain started.

t (minutes)	0	15	30	45	60	75	90	105	120
R (cm)	0	0.5	0.75	1.3	2	2.7	3.5	4.2	5

a. Explain why f is an invertible function.
b. Find $f(45)$ and interpret it in the context of the situation.
c. Find $f^{-1}(4.2)$ and interpret it in the context of the situation.

Commentary

In this task students are asked to analyze a function and its inverse when the function is given as a table of values. In addition to finding values of the inverse function from the table, they also have to explain why the given function is invertible. This can be done in two ways, either by arguing that the given function is strictly increasing, and therefore every output will come from a unique input, or by examining the given values in the table and observing that each output is attained by a unique input.

This task illustrates that a function and its inverse reverse how we can look at a situation. The original function asks "How much rain has fallen after different amounts of time?" The inverse function asks "How long did it take for a certain amount of rain to fall?"

This task could be used for instruction or as assessment. The context also lends itself to introduce the idea of inverse functions. In that case, one could ask if given an amount of rainfall it would be possible to determine how long it has been raining. If the task is used for assessment, the instructor may want to specify the domain more clearly.

Notice that the units of rainfall is given in cm, a measure of length rather than volume. However, this is how rainfall is reported most of the time, and it refers to the depth of the rain that has fallen, for example if it were captured in a right cylinder. This might be slightly confusing for the students and is worth mentioning.

Solution: 1

a. A function is invertible if every output corresponds to one and only one input. We can observe from the table that each output value occurs exactly once. Therefore, for every amount of rainfall R that is shown in the table, we can find $f^{-1}(R)$.

   We can also argue from the verbal description of the situation: Since the rainfall is described as "steady," the total amount will be an increasing function of time. Therefore, every specific amount of rainfall between 0 and 5 cm will have occurred at a unique point in time. This implies that the function is invertible.

b. We have $f(45) = 1.3$. This means that after 45 minutes the total amount of rainfall is 1.3 cm.

c. To find $f^{-1}(4.2)$ we look for the output value 4.2 in the table and find the corresponding input value. When $R = 4.2$ we see that $t = 105$. Therefore, $f^{-1}(4.2) = 105$. In the context of the situation, we are trying to find out how long it took until the total rainfall reached 4.2 cm. So we now can say that 4.2 cm of rain had fallen after 105 minutes.