Lake Algae

Alignment: MAFS.912.F-BF.1.1

On June 1, a fast growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area covered by the algae doubles every day. If it continues to grow unabated, the lake will be totally covered and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30.

a. When will the lake be covered half-way?
b. On June 26, a pedestrian who walks by the lake every day warns that the lake will be completely covered soon. Her friend just laughs. Why might her friend be skeptical of the warning?
c. On June 29, a clean-up crew arrives at the lake and removes almost all of the algae. When they are done, only 1% of the surface is covered with algae. How well does this solve the problem of the algae in the lake?
d. Write an equation that represents the percentage of the surface area of the lake that is covered in algae as a function of time (in days) that passes since the algae was introduced into the lake if the cleanup crew does not come on June 29.

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Commentary

The purpose of this task is to introduce students to exponential growth. While the context presents a classic example of exponential growth, it approaches it from a non-standard point of view. Instead of giving a starting value and asking for subsequent values, it gives an end value and asks about what happened in the past. The simple first question can generate a surprisingly lively discussion as students often think that the algae will grow linearly.

The model of algae growth given in the problem is a simplification of how algae actually grows in lakes. It is important to emphasize both that simplifying assumptions have to be made to model most problems of actual interest, and the limitations of such assumptions. In the current task, the reasonable simplifications on the algae growth model make the analysis much more accessible than more intricate models, but also lead to some subtle implausibilities (for example, the last row of the table generated in the solution to part c).

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Solution: Solution

a. Since the algae cover doubles every day, if the lake is completely covered on June 30, then one day earlier, on June 29, half of the lake was covered.

b. On June 29, half the lake was covered. On June 28 one forth of the lake was covered. On June 27 one eighth of the lake was covered. On June 26 only one sixteenth of the lake was covered by algae.

When the friend does not believe the warning, only $\frac{1}{16}$, or 6.25% of the water surface was covered, so $\frac{15}{16} = 93.75\%$ of the water surface was still open. The algae cover did not seem very dramatic at this point.

c. After the clean-up crew removes most of the algae, only 1% of the water surface is covered by algae. However, because these algae still grow just as quickly as before, the area covered still doubles every day. Therefore, after one day, 2% will be covered, after two days 4% will be covered, and so on. A table of values shows the following:

n, number of days since clean-up	percentage of lake covered with algae
0	1
1	2
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$
5	$2^5 = 32$
6	$2^6 = 64$
7	$2^7 = 128$

Of course, it is impossible to cover more than 100% of the water surface, so the last entry in the table does not make sense. But we can conclude that sometime between day 6 and 7 the entire lake will be covered by algae. The lake clean-up will be undone completely in less than a week.

d. Let t be the time (in days) since the algae was introduced into the lake, and p be the percentage of the lake that is covered by algae at time t. We know that June 1 corresponds to $t = 0$ and on June 30 (which corresponds to $t = 29$) that $p = 100$. Since the percentage of the lake that is covered in algae is doubling every day, if p_0 is the percentage of the lake covered when the algae is first introduced, we know that the equation must be of the form:

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$$p(t) = p_0 2^t.$$

It remains to find p_0. Since

$$p(29) = p_0 2^{29} = 100,$$

we conclude that

$$p_0 = \frac{100}{2^{29}} \approx 1.86 \times 10^{-7}.$$

So the equation giving the percent covered at time t (for $0 \leq t \leq 29$) is

$$p(t) = (1.86 \times 10^{-7})2^t.$$

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