

Gotham City Taxis

Task

The taxi fare in Gotham City is \$2.40 for the first $\frac{1}{2}$ mile and additional mileage charged at the rate \$0.20 for each additional 0.1 mile. You plan to give the driver a \$2 tip. How many miles can you ride for \$10?



Commentary

The purpose of this task is to give students an opportunity to solve a multi-step ratio problem that can be approached in many ways. This can be done by making a table, which helps illustrate the pattern of taxi rates for different distances traveled and with a little persistence leads to a solution which uses arithmetic. It is also possible to calculate a unit rate (dollars per mile) and use this to find the distance directly without making a table. Because the ratio in this context involves non-whole numbers, these solution approaches are aligned to 7th rather than 6th grade. The cost of the trip may be represented by an algebraic expression which can be used to find the distance which costs exactly \$10.

Because the numbers are relatively clean, the first two approaches are reasonable. However, with messier numbers, the algebraic approach has an advantage. This task could be used to help students see the value of algebraic representations. First, they could solve this problem as is, with the teacher encouraging many approaches and taking time to have students show the different ways they approached the problem. If no students use an algebraic approach, they should be encouraged to find one and discuss it as a whole class. Then a similar problem could be given with messier numbers where the algebraic solution provides an easier, more efficient way of solving the problem.

This task was adapted from problem #10 on the 2011 American Mathematics Competition (AMC) 8 Test. The responses to the multiple choice answers for the problem had the following distribution:

Choice	Answer	Percentage of Answers
(A)	3.0	10.06
(B)	3.25	10.39
(C)*	3.3	51.52
(D)	3.5	14.14
(E)	3.75	11.42
Omit	-	2.44

Of the 153,485 students who participated, 72,648 or 47% were in 8th grade, 50,433 or 33% were in 7th grade, and the remainder were less than 7th grade.



Solutions

Solution 1: Arithmetic reasoning with rates

You have \$10 to spend on the taxi ride. You want to tip \$2 so that leaves \$8. It costs \$2.40 for the first half mile, so after travelling half of a mile you have \$5.60 left. After that first half mile, it costs \$0.20 for every 0.1 mile. We can convert this to a rate of dollars per mile by multiplying by 10: it costs \$2 per mile. So to see how much further we can travel with the remaining \$5.60 we perform the division problem

$$\$5.60 \div (\$2 \text{ per mile})$$

We have $5.6 \div 2 = 2.8$. The units of dollars cancel because we have dollars in the numerator and denominator. The units of per mile in the denominator become miles (in the numerator) and so we get 2.8 miles as an answer of how much further we can travel, after the first half mile, with \$5.60. So, the total distance we can go for our \$10 is 0.5 miles and 2.8 miles for a total of 3.3 miles.

Solution 2: A Ratio Table

Here we make a table to find how far we can go in the taxi with \$10. We have listed a cost of \$2 for 0 miles indicating the tip which is added to rides of any distance. The first half mile is also listed as this is charged at a special rate of \$2.40. After this, each half mile increment costs an additional \$1 (5 times 20 cents per 0.1 mile). Here is a table:

Distance travelled in miles	Cost in dollars
0	2.0
0.5	4.4
1.0	5.4
1.5	6.4
2.0	7.4
2.5	8.4
3.0	9.4
3.5	10.4

From this we see that the answer must be between 3.0 and 3.5 miles. We can now make a new table:



Distance travelled in miles	Cost in dollars
3.0	9.4
3.1	9.6
3.2	9.8
3.3	10.0

So with \$10 it will be possible to travel 3.3 miles in Gotham City. Note that these tables are not ratio tables: because of the tip and the different rate for the first half mile, the ratio between the distance travelled and the cost of the taxi ride is not constant.

Solution 3: Making an equation

The minimum charge is for $\frac{1}{2}$ mile, so let x be the number of miles we can travel in the taxi past the first half mile. Each additional tenth of a mile costs 20 cents so each additional mile costs \$2. So it costs $2x$ dollars to travel the x miles beyond the first half mile. Since the first half mile costs \$2.40 and the tip will be \$2.00, the total cost of the trip is $2x + 2.40 + 2.00$ or

$$2x + 4.40$$

If the cost of the trip is \$10, then

$$2x + 4.40 = 10.$$

Subtracting 4.40 from both sides gives

$$2x = 5.60.$$

Dividing both sides by 2 gives

$$x = 2.80.$$

To find the total distance traveled, we need to find $x + 0.5$, and $2.8 + 0.5 = 3.3$.

Thus, with \$10 we can travel 3.3 miles in the taxi.