

Graphs of Compositions

Task

For each function in this task, assume the domain is the largest set of real numbers for which the function value is a real number.

Let f be the function defined by $f(x) = x^2$. Let g be the function defined $g(x) = \sqrt{x}$.

- Sketch the graph of $y = f(g(x))$ and explain your reasoning.
- Sketch the graph of $y = g(f(x))$ and explain your reasoning.

Commentary

This task addresses an important issue about inverse functions. In this case the function f is the inverse of the function g but g is not the inverse of f unless the domain of f is restricted.

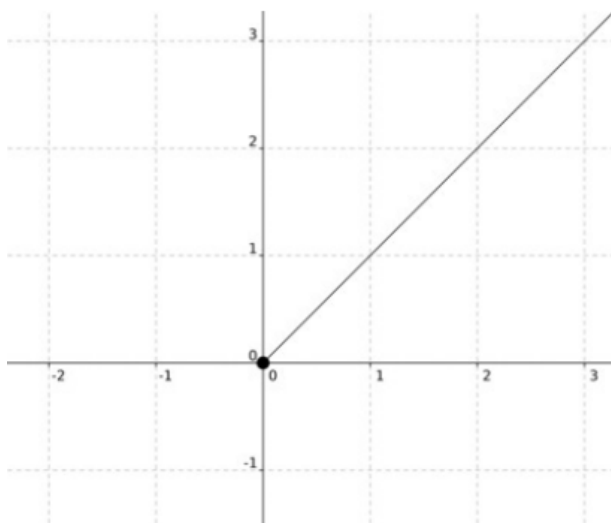


Solution

a. We have

$$f(g(x)) = (\sqrt{x})^2 = x.$$

The domain of \sqrt{x} is all non-negative real numbers and so the graph starts at the point $(0,0)$ and then includes all positive values of x in its domain:



b. We have

$$g(f(x)) = \sqrt{x^2}.$$

The square root $\sqrt{}$ takes non-negative real numbers as input and gives non-negative numbers as output. So $\sqrt{x^2} = x$ if x is non-negative and $\sqrt{x^2} = -x$ if x is negative. In other words $g(f(x)) = |x|$ the absolute value function.

