Ants versus humans

The average mass of an adult human is about 65 kilograms while the average mass of an ant is approximately 4×10^{-3} grams. The total human population in the world is approximately 6.84 billion, and it is estimated there are currently about 10,000 trillion ants alive.[1]

Based on these values, how does the total the total mass of all living ants compare to the total mass of all living humans?

Commentary

This task requires students to work with very large and small values expressed both in scientific notation and in decimal notation (standard form). In addition, students need to convert units of mass. The solution below converts the mass of humans into grams; however, we could just as easily convert the mass of ants into kilograms. Students are unable to go directly to a calculator without taking into account all of the considerations mentioned above. Even after converting units and decimals to scientific notation, students should be encouraged to use the structure of scientific notation to regroup the products by extending the properties of operations and then use the properties of exponents to more fluently perform the calculations involved rather than rely heavily on a calculator. In particular, teachers could encourage students to estimate the ratios of the two masses before resorting to calculator use.

This task provides an opportunity to discuss the accuracy of these calculations and an appropriate level of precision. For example, it is obviously not possible to count the total number of ants in the world at any given moment, nor is it feasible to weigh every adult human. Similarly, the numbers in the task statement make heavy use of "averages" (implicit and unstated is that these averages are mean values, bringing an opportunity to discuss precision in mathematical terminology), a process which inherently raises questions -- were children included in the average human mass calculation? How would their exclusion affect the conclusion? Questions of this form abound, and their analysis fosters critical thinking skills.

[1] Holldobler, B., E. Wilson, *Journey to the Ants: A Story of Scientific Exploration* (London, England: The Belknap Press of Harvard University, 1994).

Solution

We are told the total number of ants in the world is about 10,000 trillion or

$$\begin{aligned} 10{,}000\times10^{12} &= 10^4\times10^{12} \\ &= 10^{16} \end{aligned}$$

ants. In addition, the average mass of a single ant is 4×10^{-3} grams. Thus, the approximate total mass of all ants in the world is

$$\begin{aligned} (4\times10^{-3}\text{g})(10^{16}) &= 4\times10^{-3+16}\text{g} \\ &= 4\times10^{13}\text{g} \end{aligned}$$

The mass for humans is given in kilograms while the mass for ants is in grams. We convert the unit of mass for a human to grams as follows,

$$\begin{aligned} (65\text{kg})\left(\frac{10^3\text{g}}{1\text{kg}}\right) &= (65\times10^3)\text{g} \\ &= (6.5\times10^1)(10^3)\text{g} \\ &= 6.5\times10^{1+3}\text{g} \\ &= 6.5\times10^4\text{g} \end{aligned}$$

Since there are 6.84 billion humans on earth, the total mass of all humans on earth can be approximated as

$$\begin{aligned} (6.5\times10^4\text{g})(6.84\text{ billion}) &= (6.5\times10^4\text{g})(6.84\times10^9) \\ &= (6.5\times6.84)(10^4\times10^9)\text{g} \\ &= 44.46\times10^{4+9}\text{g} \\ &= (4.446\times10^1)\times10^{13}\text{g} \\ &= 4.446\times10^{14}\text{g} \end{aligned}$$

Thus, the total mass of all humans in the world is greater than the total mass of all ants in the world. In fact, the calculations above show the total mass of all humans in the world is about 10 times the total mass of all ants.