Building a quadratic function from $f(x) = x^2$

Task
Suppose $f(x) = x2$ where x can be any real number.

a. Sketch a graph of the function f.

b. Sketch a graph of the function g given by

$$g(x) = f(x) + 2.$$

How do the graphs of f and g compare? Why?

c. Sketch a graph of the function h given by

$$h(x) = -2 \cdot f(x).$$

How do the graphs of f and h compare? Why?

d. Sketch a graph of the function p given by

$$p(x) = f(x + 2).$$

How do the graphs of f and p compare? Why?

Commentary
This is the first of a series of task aiming at understanding the quadratic formula in a geometric way in terms of the graph of a quadratic function. Here the student works with an explicit function and studies the impact of

- An additive scaling of f
- A multiplicative scaling of f
- A linear change of variables applied to f

The students can either sketch the graphs by hand or use graphing calculators. The important part of this task is recognizing the impact of the different transformations on the graphs. An alternative way to proceed with this task would be to give the students the graphs and the list of functions f, g, h, p and ask them to identify the functions with their graphs and explain. This task is appropriate for either instruction or assessment.

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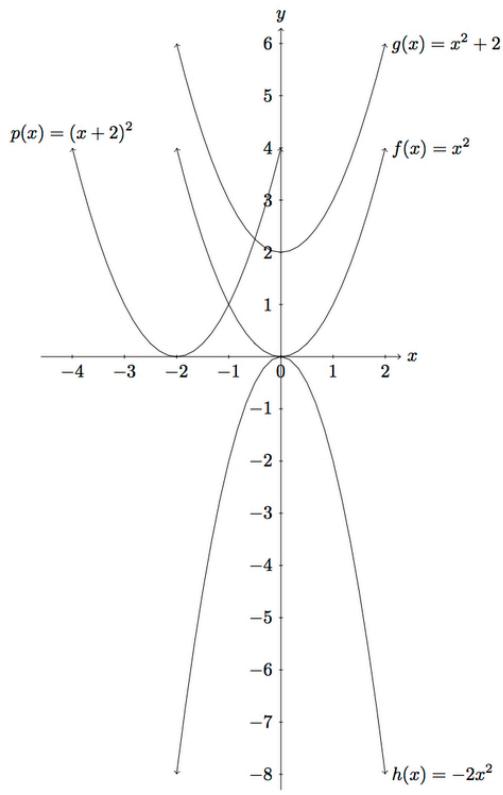
Solution

The graphs of f, g, h, and p are shown (within the range of $-2 \leq x \leq 2$) below together to reveal their geometric relationship:



a. Since $f(x) = x^2$ the graph of f is a parabola. It opens upward and its vertex is at $(0, 0)$.

b. Looking at the two points $(x,\ f(x))$ and $(x,\ g(x))$ we have

$$(x, g(x)) = (x, f(x) + 2)$$

for every value of x. So for each point $(x, f(x))$ on the graph of f, there is a corresponding point $(x, f(x) + 2)$ on the graph of g, located two units above $(x, f(x))$. This means that the graph of g is the same as the graph of f, displaced 2 units upward.

c. The point $(x, h(x))$ is the same as $(x, -2f(x))$ so the graph of h is the same as the reflection of the graph of $2f$ about the x-axis. So the values of f are first doubled, exaggerating the slope of the graph, and then the graph is reflected about the x-axis.

d. The point $(x, p(x))$ on the graph of p is the same as $(x, f(x + 2))$ which is two units to the right of the point $(x, f(x))$ on the graph of f. So the graph of p looks like the graph of f, translated two units to the left: put in other words, $p(x - 2) = f(x)$ so the point $(x - 2, f(x))$ is on the graph of p while $(x, f(x))$ is the corresponding point of the graph off and so the graph of p is the graph of f shifted two units to the left.

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