# The Missing Coefficient

**MAFS.912.A-APR.2.2**

## Task

Consider the polynomial function

$$P(x) = x^4 - 3x^3 + ax^2 - 6x + 14,$$

where $a$ is an unknown real number. If $(x - 2)$ is a factor of this polynomial, what is the value of $a$?

## Commentary

The purpose of this task is to emphasize the use of the Remainder Theorem (a discussion of which should obviously be considered as a prerequisite for the task) as a method for determining structure in polynomial in equations, and in this particular instance, as a replacement for division of polynomials.

Indeed, one possible solution path is to use polynomial division to divide $P(x)$ by $(x - 2)$ and determine the remainder in terms of $a$, and then solve for a by setting the remainder equal to zero. However, the division operation becomes unwieldy with the unknown parameter $a$ in play. A more straightforward approach is to use the Remainder Theorem (MAFS.912.A-APR.2.2), which states that if $(x - 2)$ is to be a factor of $P(x)$, then $P(2)$ must equal zero.

**Solution**

By the Remainder Theorem, if $(x-2)$ is a factor of $P(x)$, then $P(2)$ must equal zero. Therefore, we must have

$$P(2) = 16 - 3 \cdot 8 + a \cdot 4 - 6 \cdot 2 + 14 = 0.$$

Simplifying, we find that $4a - 6 = 0$, and thus $a = 3/2$.