Fuel Efficiency

Sadie has a cousin Nanette in Germany. Both families recently bought new cars and the two girls are comparing how fuel efficient the two cars are. Sadie tells Nanette that her family's car is getting 42 miles per gallon. Nanette has no idea how that compares to her family's car because in Germany mileage is measured differently. She tells Sadie that her family's car uses 6 liters per 100 km. Which car is more fuel efficient?

Licensed by Illustrative Mathematics under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License

Commentary

The problem requires students to not only convert miles to kilometers and gallons to liters but they also have to deal with the added complication of finding the reciprocal at some point. In the USA we use distance per unit volume to measure fuel efficiency but in Europe we use volume per unit distance. Furthermore, the unit of distance is not simply 1 km but rather 100 km.

The required computation is not particularly long, but requires a firm understanding of the concepts and skills of unit conversion. The task would therefore be suitable for individual assessment, or alternatively, this task could be used as an activity where students work in groups and discuss their strategies. The problem also has elements of a modeling task since students have to decide on a solution method and find the appropriate conversion factors. Most likely, they have not been presented with a similar task before, so there won't be a clear solution path to follow. To solve the problem it is particularly helpful to follow the units of the quantities involved.

Licensed by Illustrative Mathematics under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License

Solution: mpg to liters per 100 km

We need to convert miles per gallon to liters per 100 km. So we somehow have to get from miles to km and from gallons to liters. A quick Google search finds that 1 mile = 1.609 km and 1 gallon = 3.79 liters.

We can start by converting miles per gallon into km per liter.

$$\begin{aligned}\frac{42 \text{ miles}}{1 \text{ gallon}} &= \frac{42 \text{ miles}}{1 \text{ gallon}} \cdot \frac{1.609 \text{ km}}{1 \text{ mile}} \cdot \frac{1 \text{ gallon}}{3.79 \text{ liters}} \\ &= \frac{67.578 \text{ km}}{3.79 \text{ liters}}\end{aligned}$$

Since we want to find the answer in liters per unit distance, we need to take the reciprocal and convert the units in the denominator into 100 km to finish the computation:

$$\begin{aligned}\frac{3.79 \text{ liters}}{67.578 \text{ km}} &= 0.0561 \text{ liters per km} \\ &= 5.61 \text{ liters per 100 km}\end{aligned}$$

This means that 42 miles per gallon is equivalent to 5.61 liters per 100 km, and Sadie's family's car is slightly more fuel efficient than Nanette's family's car.

Solution: liters per 100 km to mpg

As an alternative solution, we would start with 6 liters per 100 km and converted it to miles per gallon:

$$\frac{6 \text{ liters}}{100 \text{ km}} = \frac{0.06 \text{ liters}}{1 \text{ km}}$$

This is equivalent to

$$\frac{1 \text{ km}}{0.06 \text{ liters}} = \frac{1 \text{ km}}{0.06 \text{ liters}} \cdot \frac{1 \text{ mile}}{1.609 \text{ km}} \cdot \frac{3.79 \text{ liters}}{1 \text{ gallon}} = \frac{39.26 \text{ miles}}{1 \text{ gallon}}.$$

Nanette's family's car gets 39.26 miles per gallon, which is slightly worse than Sadie's family's car at 42 miles per gallon.

Licensed by Illustrative Mathematics under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License