Forms of Exponential Expressions

Task

Four physicists describe the amount of a radioactive substance, Q in grams, left after t years:

i. $Q = 300e^{-0.0577t}$

ii. $Q = 300(1/2)^{t/12}$

iii. $Q = 300 \cdot 0.9439^{t}$

iv. $Q = 252.290 \cdot 0.9439^{t-3}$

a. Show that the expressions describing the radioactive substance are all equivalent (using appropriate rounding).

b. What aspect of the decay of the substance does each of the formulas highlight?

IM Commentary

There are many different ways to write exponential expressions that describe the same quantity, in this task the amount of a radioactive substance after t years. Depending on what aspect of the context we need to investigate, one expression of the quantity may be more useful than another. This task contrasts the usefulness of four equivalent expressions. Students first have to confirm that the given expressions for the radioactive substance are equivalent. Then they have to explain the significance of each expression in the context of the situation.

The task can be used for assessment, practice or even to motivate via an in-class discussion why it is useful to rewrite exponential expressions in equivalent forms.

Solution

a. Using properties of exponents we can transform the expressions that describe the amount of the radioactive substance into each other. We have

   i. $300e^{-0.0577t} = 300(e^{-0.0577})^t = 300 \cdot 0.9439^t$.

   Similarly,

   ii. $300 \cdot (1/2)^{t/12} = 300((1/2)^{1/12})\ t = 300 \cdot 0.9439^t$.

   Finally,

   iv. $252.290 \cdot 0.9439^{t-3} = 252.290 \cdot 0.9439^{-3} \cdot 0.9439^t = 300 \cdot 0.9439^t$.

b. The first three formulas show that the initial amount of the substance is 300 grams.

   i. This formula lets us read off the fact that the continuous decay rate is 5.77 %. (Note: The substance decays at a rate that is proportional to the amount present at any time and the constant of proportionality is 0.0577.)

   ii. If we substitute $t = 12$ we get $Q = 300 \cdot (1/2)$. Therefore, this formula shows that the half-life of the substance is 12 years.

   iii. Since $1 - 0.9439 = 0.0561$ we see from this formula that the annual decay rate is 5.61 %.

   iv. In addition to the annual decay rate, this formula also shows that when $t = 3$ we have $Q = 252.290$. This means that after 3 year there are 252.290 grams of the substance left.