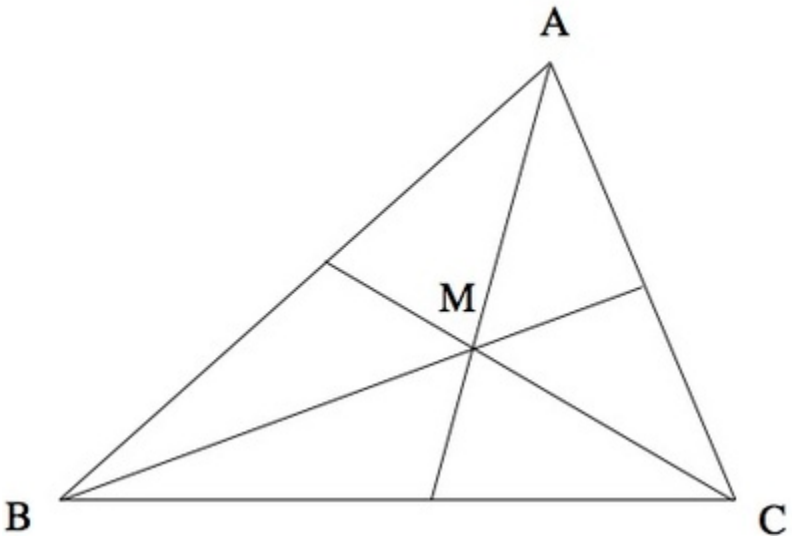
Find the Angle

In triangle ΔABC, point M is the point of intersection of the bisectors of angles $\angle BAC$, $\angle ABC$, and $\angle ACB$. The measure of $\angle ABC$ is 42°, and the measure of $\angle BAC$ is 64°. What is the measure of $\angle BMC$?



This task adapted from a problem published by the Russian Ministry of Education.

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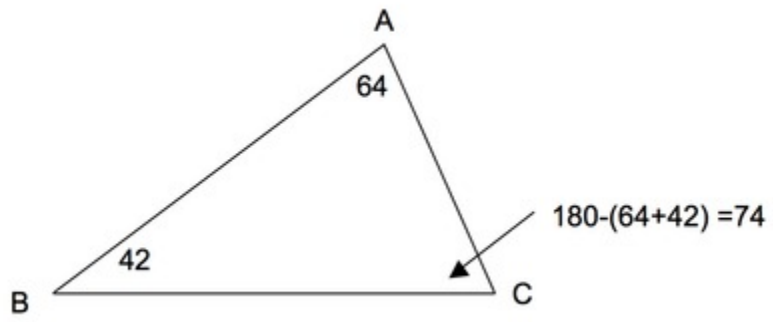
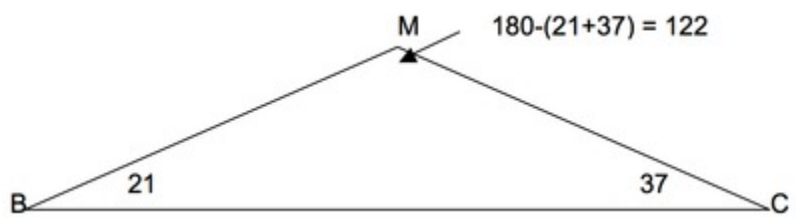
Solution

All angle measurements are in degrees.

The solution is obtained by applying the Triangle Sum Theorem twice. First apply it to the triangle *ABC* to find the measure of angle *ACB*. This angle has measure 180° − (64° + 42°) = 180° − (106°) = 74°:



Now consider the triangle *BMC*. Since the segment *BM* bisects the angle *ABC* of the triangle, we have the measure of angle *MBC* is half the measure of angle *ABC*, which is half of 42°, or 21°. Similarly, the measure of angle *MCB* is half of angle *ACB*, which is half of 74°, which is 37°. Now use the Triangle Sum Theorem on the triangle *BMC* to find that the measure of angle *BMC* is 180° − (37° + 21°) = 180° − 58° = 122°:



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