

Profit of a Company

The profit that a company makes selling an item (in thousands of dollars) depends on the price of the item (in dollars). If p is the price of the item, then three equivalent forms for the profit are:

$$\text{Standard form: } -2p^2 + 24p - 54$$

$$\text{Factored form: } -2(p - 3)(p - 9)$$

$$\text{Vertex form: } -2(p - 6)^2 + 18.$$

Which form is most useful for finding

- a. The prices that give a profit of zero dollars?
- b. The profit when the price is zero?
- c. The price that gives the maximum profit?



Commentary

This task compares the usefulness of different forms of a quadratic expression. Students have to choose which form most easily provides information about the maximum value, the zeros and the vertical intercept of a quadratic expression in the context of a real world situation. Rather than just manipulating one form into the other, students can make sense out of the structure of the expressions.

(From *Algebra: Form and Function*, McCallum et al., Wiley 2010)

Solution: Structure of quadratic expression

- The factored form gives the values of p that make the profit zero. Since factored form is $-2(p - 3)(p - 9)$ the profit is zero when $p = 3$ or $p = 9$. The company breaks even if the price charged for the product is \$3 or \$9.
- The standard form is the easiest one to use to find the profit when the price is zero. Substituting $p = 0$ into the standard form $-2p^2 + 24p - 54$ we see that the profit is -54 (in thousands of dollars) when the price is zero. If the company gives the product away for free, it loses \$54,000.
- The vertex form shows us what price maximizes profit. From the expression $-2(p - 6)^2 + 18$, we see that the maximum profit is 18 thousand dollars, and it occurs when $p = 6$. The company should charge a price of \$6 for this product.

