Increasing or Decreasing? Variation 2

Task

Consider the expression:

$$\frac{R_1 + R_2}{R_1 R_2}$$

where R_1 and R_2 are positive.

Suppose we increase the value of R_1 while keeping R_2 constant. Find an equivalent expression whose structure makes clear whether the value of the expression increases, decreases, or stays the same.

Commentary

The purpose of this task is to help students see manipulation of expressions as an activity undertaken for a purpose.

Variation 1 of this task presents a related more complex expression already in the correct form to answer the question.

The expression arises in physics as the reciprocal of the combined resistance of two resistors in parallel. However, the context is not explicitly considered here.

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Solution

We rewrite:

$$\frac{R_1 + R_2}{R_1 R_2}$$

in the form:

$$\frac{R_1}{R_1 R_2} + \frac{R_2}{R_1 R_2} = \frac{1}{R_2} + \frac{1}{R_1}$$

Now the bigger a (positive) number is, the smaller its reciprocal is, and so the value of this expression decreases as R_1 increases.

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