Mixing Fertilizer

A landscaping company prepares two mixtures of fertilizer.

a. Mixture A contains 5 liters of liquid fertilizer and 50 liters of water. What is the concentration of fertilizer (by volume) in Mixture A?

b. Mixture B contains 10 liters of liquid fertilizer and 50 liters of water. What is the concentration of fertilizer (by volume) in Mixture B?

c. The volume of the fertilizer in Mixture B is twice the volume of the fertilizer in Mixture A. Why isn't the concentration of fertilizer (by volume) in Mixture B double that of Mixture A?

d. Write an expression for the concentration of fertilizer (by volume) in Mixture A in terms of the volume of the fertilizer in Mixture A, F_a, and the volume of the water, W.

e. Explain your answer to (c) in terms of F_a and W.

Commentary

The problem deals with a rational expression which is built up from operations arising naturally in a context: adding the volumes of the fertilizer and the water, and dividing the volume of the fertilizer by the resulting sum. Thus, it encourages students to see the expression as having meaning in terms of numbers and operations, rather than as an abstract arrangement of symbols. This helps them think though how a change in one of the quantities affects the value of the expression, and to see the difference algebraically between doubling one of the variables and doubling the value of the expression.

Much of this problem addresses issues from middle school or Algebra I. But the use of rational expressions in the last part pushes it into Algebra II in many curricula.

Solution: Mixing Fertilizer

a. The volume of the fertilizer is 5 liters and the volume of the water is 50 liters so the total volume of the mixture is $5+50=55$ liters. Thus the concentration of the fertilizer (by volume) in Mixture A is:

$$\frac{\text{volume of the fertilizer}}{\text{volume of the mixture}} = \frac{5}{5+50} = \frac{1}{11} \approx .0909 = 9.09\%$$

b. The volume of the fertilizer is 10 liters and the volume of the water is 50 liters so the total volume of the mixture is 60 liters. Thus the concentration of the fertilizer (by volume) in Mixture B is:

$$\frac{\text{volume of the fertilizer}}{\text{volume of the mixture}} = \frac{10}{10+50} = \frac{1}{6} \approx .167 = 16.7\%.$$

c. Since doubling the volume of the fertilizer does double the value of the numerator in the concentration fraction, some might think that the concentration fraction for Mixture B is twice that of Mixture A. But the increase in fertilizer volume in Mixture B also increases the value of one of the terms of the sum in the denominator. Thus, the concentration of Mixture B is not double that of Mixture A.

d.
$$\frac{\text{volume of the fertilizer}}{\text{volume of the mixture}} = \frac{F_a}{F_a+W}$$

e. The fertilizer concentration in Mixture B is given by

$$\frac{2F_a}{2F_a+W}.$$

This is not equivalent to

$$2\left(\frac{F_a}{F_a+W}\right) = \frac{2F_a}{F_a+W},$$

as the computations in (a) and (b) demonstrate.