Delivery Trucks

A company uses two different-sized trucks to deliver sand. The first truck can transport x cubic yards, and the second y cubic yards. The first truck makes S trips to a job site, while the second makes T trips. What quantities do the following expressions represent in terms of the problem's context?

a. $S+T$

b. $x+y$

c. $xS+yT$

d. $\dfrac{xS+yT}{S+T}$

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Commentary

The task has students look for structure in algebraic expressions related to a context, and asks them to relate that structure to the context. In addition, the task is illustrative of several of the mathematical practice standards. In this task, students have to be quite fluent in moving back and forth between the symbolic representation of a quantity and its interpretation in terms of the problem's context. While the task falls just short of explicitly requires students to reason about units, the requirement is present nonetheless -- students have to, for example, interpret xS as a measurement of cubic yards per trip times a number of trips, resulting in a quantity whose units are simply cubic yards.

Adapted from Algebra: Form and Function, McCallum et al, Wiley, 2010.

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Solution: Solution

a. S is the number of trips the first truck makes to a job site, and T is the number of trips the second truck makes to a job site. It follows that

$$S+T = \text{the total number of trips both trucks make to a job site}$$

b. We know that x and y are the amount of sand, in cubic yards, that the first and second truck can transport, respectively. Then

$$x+y = \text{the total amount of sand that both trucks can transport together}$$

In other words, the company can transport $x+y$ cubic yards of sand in a single trip using both trucks.

c. We can think of $xS+yT$ in separate terms. The first term, xS, multiplies x, the amount of sand the first truck can transport, by S, the number of trips the first truck makes to a job site. This means

$$xS = \text{the total amount of sand being delivered to a job site by the first truck}$$

In the second term, y, the amount of sand the second truck can transport, is being multiplied by T, the number of trips the second truck makes. This means

$$yT = \text{the total amount of sand being delivered to a job site by the second truck}$$

We then have that

$$xS+yT = \text{the total amount of sand (in cubic yards) being delivered to a job site by both trucks.}$$

d. From part (c), we know that $xS+yT$ is the total amount of sand, in cubic yards, being delivered to a job site. We also know from part (a) that $S+T$ is the number of total trips being made to a job site. By dividing $xS+yT$ by $S+T$, we are averaging out the amount of sand being transported over the total number of trips. So,

$$\frac{xS+yT}{S+T} = \text{the average amount of sand being transported per trip.}$$

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