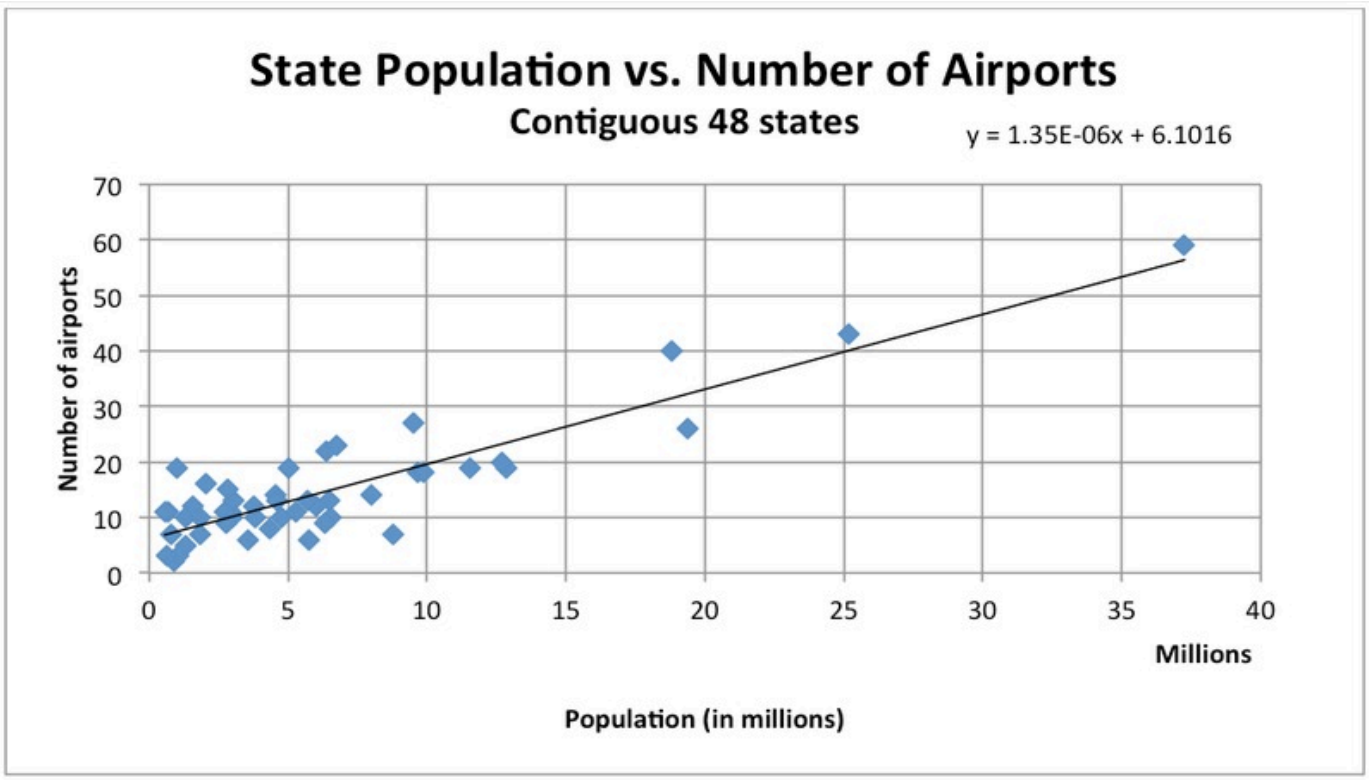
Scatter Plot: US Airports

The scatter plot below shows the relationship between the number of airports in a state and the population of that state according to the 2010 Census. Each dot represents a single state.



a. How would you characterize the relationship between the number of airports in a state and the state's population? (Select one):
   i. The variables are positively associated; states with higher populations tend to have fewer airports.
   ii. The variables are negatively associated; states with higher populations tend to have fewer airports.
   iii. The variables are positively associated; states with higher populations tend to have more airports.
   iv. The variables are negatively associated; states with higher populations tend to have more airports.
   v. <u>The variables are not associated.</u>

LaToya uses the function $y = (1.35\times10^{-6})x + 6.1$ to model the relationship between the number of airports, *y* and the population in a state, *x*.

b. How many airports does LaToya's model predict for a state with a population of 30 million people? [_____].

c. What does the number 6.1 that appears in LaToya's function mean in the context of airports vs. populations? (Select one.)

i. The average number of airports in a state is 6.1.
ii. The median number of airports in a state is 6.1.
iii. The model predicts a population of 6.1 people in a state with no airports.
iv. The model predicts 6.1 airports in a state with no people.
v. The model predicts that 6.1 states have no airports.
vi. The model predicts 6.1 more airports, on average, for each additional person in a state.
vii. The model predicts 6.1 fewer airports, on average, for each additional person in a state.
viii. The number 6.1 cannot be interpreted in this context.

d. What does the number (1.35×10^{-6}) that appears in LaToya's function mean in the context of airports vs. populations? (Select one.)

i. The average number of airports in a state is (1.35×10^{-6})
ii. The median number of airports in a state is (1.35×10^{-6})
iii. The model predicts (1.35×10^{-6}) airports in a state with no people.
iv. The model predicts (1.35×10^{-6}) people in a state with no airports.
v. The model predicts that (1.35×10^{-6}) states have no airports.
vi. The model predicts (1.35×10^{-6}) more airports, on average, for each additional person in a state.
vii. The model predicts (1.35×10^{-6}) fewer airports, on average, for each additional person in a state.
viii. The number (1.35×10^{-6}) cannot be interpreted in this context.

e. Fill in the following newspaper headline based on this relationship:

On average, a state in the contiguous 48 US states has 1 additional airport for every _____ additional people.

Commentary

This task is part of a joint project between Student Achievement Partners and Illustrative Mathematics to develop prototype machine-scorable assessment items that test a range of mathematical knowledge and skills.

Purpose

This is one of two assessment tasks illustrating the similarities and differences between the 8th grade standards in Functions and in Statistics and Probability. The first, Mail Truck, involves a situation that can be modeled exactly with a linear function. The second, US Airports, uses a linear function to model a relationship between two quantities that show statistical variation and do not have an exact linear relationship.

In US Airports, each additional person in the state does not directly correspond to a portion of an airport, but the relationship can be modeled using a linear association, and the model can be used to make predictions about the number of airports in states with a given population. In Mail Truck, each additional day of driving does correspond to exactly the same increase in the number of miles put onto the truck each day.

Cognitive Complexity

Mathematical Content

This task involves constructing a linear function and interpreting its parameters in a context. Thus, this task has a medium level of complexity.

Mathematical Practice

The task asks students to reason abstractly and quantitatively and directly assesses component skills related to mathematical modeling, namely, interpreting mathematical objects in contexts.

Linguistic Demand

This context in this task requires students to interpret the mathematics in this context, so has a high level of linguistic complexity.

Stimulus Material

The stimulus material is not complex.

Response Mode

The interface is not complex.

Solution

a. (iii)
b. 46.6 airports
c. (iv)
d. (vi)
e. about 700 thousand or 700,000 or 740,000

This is a 4-point item.

3 points if a student misses one part.

2 points if a student misses 2 parts

1 point if a student misses 3 parts.

0 points if a student misses 4 or 5 parts.