Standard #: MAFS.912.F-LE.1.2

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★

Grade: 912

Cluster: Construct and compare linear, quadratic, and exponential models and solve problems. (Algebra 1 - Supporting Cluster) (Algebra 2 - Supporting Cluster) -
Clusters should not be sorted from Major to Supporting and then taught in that order. To do so would strip the coherence of the mathematical ideas and miss the opportunity to enhance the major work of the grade with the supporting clusters.

Date Adopted or Revised: 02/14

Content Complexity Rating: Level 2: Basic Application of Skills & Concepts - More Information

Date of Last Rating: 02/14

Status: State Board Approved

Assessed: Yes

TEST ITEM SPECIFICATIONS

Item Type(s): This benchmark may be assessed using: EE item(s)

Also assesses:

MAFS.912.F-BF.1.1

MAFS.912.F-IF.1.3

Assessment Limits :
In items where the student must write a function using arithmetic operations or by composing functions, the student should have to generate the new function only.

In items where the student constructs an exponential function, a geometric sequence, or a recursive definition from input-output pairs, at least two sets of pairs must have consecutive inputs.

In items that require the student to construct arithmetic or geometric sequences, the real-world context should be discrete.

In items that require the student to construct a linear or exponential function, the real-world context should be continuous.

Calculator :
Neutral

Clarification :
Students will write a linear function, an arithmetic sequence, an exponential function, or a geometric sequence when given a graph that models a real-world context.

Students will write a linear function, an arithmetic sequence, an exponential function, or a geometric sequence when given a verbal description of a real-world context.

Students will write a linear function, an arithmetic sequence, an exponential function, or a geometric sequence when given a table of values or a set of ordered pairs that model a real-world context.

Students will write an explicit function, define a recursive process, or complete a table of calculations that can be used to mathematically define a real-world context.

Students will write a function that combines functions using arithmetic operations and relate the result to the context of the problem.

Students will write a function to model a real-world context by composing functions and the information within the context.

Students will write a recursive definition for a sequence that is presented as a sequence, a graph, or a table.

Stimulus Attributes :
For F-LE.1.2 and F-BF.1.1, items should be set in a real-world context.

For F-IF.1.3, items may be set in a mathematical or real-world context.

Items must use function notation.

In items where the student builds a function using arithmetic operations or by composition, the functions may be given using verbal descriptions, function notation or as equations.

Response Attributes :
For F-BF.1.1b and c, the student may be asked to find a value.

For F-LE.1.2 and F-BF.1.1, items may require the student to apply the basic modeling cycle.

In items where the student writes a recursive formula, the student may be expected to give both parts of the formula.

The student may be required to determine equivalent recursive formulas or functions.

Items may require the student to choose an appropriate level of accuracy.

Items may require the student to choose and interpret the scale in a graph.

Items may require the student to choose and interpret units.

SAMPLE TEST ITEMS (1)

Test Item #: Sample Item 1
Question:
A study estimates that the cost of tuition at a university will increase by 2.8% each year. The cost of tuition at the university in 2015 was $33,741.

The function, B(x), models the estimated tuition cost, where x is the number of years since 2015.

Click on the blank to enter an expression that completes the function B(x).

Difficulty: N/A
Type: EE: Equation Editor

Related Courses

Course Number	Course Title
1200310:	Algebra 1 (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200320:	Algebra 1 Honors (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200370:	Algebra 1-A (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200400:	Intensive Mathematics (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1207310:	Liberal Arts Mathematics (Specifically in versions: 2014 - 2015, 2015 and beyond (current))

7912080:	Access Algebra 1A (Specifically in versions: 2014 - 2015, 2015 - 2018, 2018 - 2019, 2019 and beyond (current))
1200315:	Algebra 1 for Credit Recovery (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200375:	Algebra 1-A for Credit Recovery (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
7912075:	Access Algebra 1 (Specifically in versions: 2014 - 2015, 2015 - 2018, 2018 - 2019, 2019 and beyond (current))

Related Access Points

Access Point

Access Points Number	Access Points Title
MAFS.912.F-LE.1.AP.2a:	Select the graph, the description of a relationship or two input-output pairs of linear functions.

Related Resources

Problem-Solving Task

Name	Description
Algae Blooms:	In this example, students are asked to write a function describing the population growth of algae. It is implied that this is exponential growth.
Basketball Bounces, Assessment Variation 1:	Students are asked to select the best model for a given context and use the model to make predictions. The purpose of this task is to assess aspects of students' modeling skills in the context of standards: F-LE.1.1.c Distinguish between situations that can be modeled with linear functions and with exponential functions; Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F-LE.1.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). This task asks students to analyze a set of data from a physical context, choose a model that reasonably fits the data, and use the model to answer questions about the physical context. This variant of the task is scaffolded; for a less scaffolded version, see F-LE Basketball Bounces, Assessment Variation 2.
Basketball Bounces, Assessment Variation 2:	This task asks students to analyze a set of data from a physical context, choose a model that reasonably fits the data, and use the model to answer questions about the physical context. This variant of the task is not scaffolded; for a more scaffolded version, see F-LE Basketball Bounces, Assessment Variation 1.
Basketball Rebounds:	This task involves a fairly straightforward decaying exponential. Filling out the table and developing the general formula is complicated only by the need to work with a fraction that requires decisions about rounding and precision.
Carbon 14 Dating in Practice II:	This problem introduces the method used by scientists to date certain organic material. It is based not on the amount of the Carbon 14 isotope remaining in the sample but rather on the ratio of Carbon 14 to Carbon 12. This ratio decreases, hypothetically, at a constant exponential rate as soon as the organic material has ceased to absorb Carbon 14, that is, as soon as it dies.
Do two points always determine a linear function?:	This problem complements the problem "Do two points always determine a linear function?" There are two constraints on a pair of points R1 and R2 if there is an exponential function f(x) = ae^bx whose graph contains R1 and R2. First, the y-coordinates of R1 and R2 cannot have different signs, that is it cannot be that one is positive while the other is negative. This is because the function g(x) = ex takes only positive values. Consequently, f(x) = ae^bx cannot take both positive and negative values. Furthermore, the only way aebx can be zero is if a = 0 and then the function is linear rather than exponential. As long as the y-coordinates of R1 and R2 are non-zero and have the same sign, there is a unique exponential function f(x) = ae^bx whose graph contains R1 and R2.
Do two points always determine an exponential function?:	This problem complements the problem "Do two points always determine a linear function?" There are two constraints on a pair of points R1 and R2 if there is an exponential function f(x) = ae^bx whose graph contains R1 and R2.
Doubling your money:	This task asks students to write equations to predict how much money will be in a savings account at the end of each year, based on different factors like interest rates.
Population and Food Supply:	In this task students use verbal descriptions to construct and compare linear and exponential functions and to find where the two functions intersect (F-LE.2, F-LE.3, A-REI.11).
Rumors:	This problem is an exponential function example that uses the real-world problem of how fast rumors spread.
Sandia Aerial Tram:	The task provides an opportunity for students to engage in detailed analysis of the rate of change of the elevation.
Two Points Determine an Exponential Function I:	This problem solving task asks students to graph a function and find the values of points on a graph.
Two Points Determine an Exponential Function II:	This problem solving tasks asks students to find the values of points on a graph.
What functions do two graph points determine?:	This problem solving task challenges students to find the linear, exponential and quadratic functions based on two points.

Original Student Tutorial

Name	Description
Creating Exponential Functions:	By the end of this tutorial, you should be able construct an exponential function from a graph, from a table of values, and from a description of a relationship in the real world.

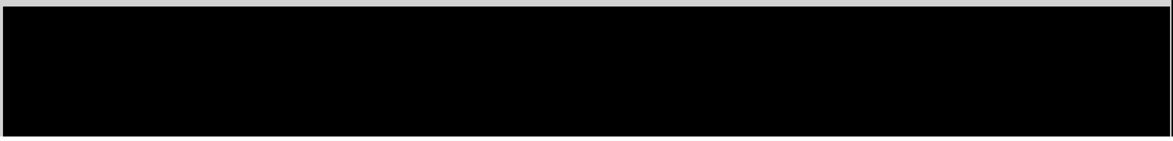
Virtual Manipulative

Name	Description
Data Flyer:	Using this virtual manipulative, students are able to graph a function and a set of ordered pairs on the same coordinate plane. The constants, coefficients, and exponents can be adjusted using slider bars, so the student can explore the affect on the graph as the function parameters are changed. Students can also examine the deviation of the data from the function. This activity includes supplemental materials, including background information about the topics covered, a description of how to use the application, and exploration questions for use with the java applet.
Introduction to Functions:	This lesson is designed to introduce students to functions as rules and independent and dependent variables. The lesson provides links to discussions and activities that motivate the idea of a function as a machine as well as proper terminology when discussing functions. Finally, the lesson provides links to follow-up lessons designed for use in succession to the introduction of functions.
Linear Function Machine:	In this activity, students plug values into the independent variable to see what the output is for that function. Then based on that information, they have to determine the coefficient (slope) and constant(y-intercept) for the linear function. This activity allows students to explore linear functions and what input values are useful in determining the linear function rule. This activity includes supplemental materials, including background information about the topics covered, a description of how to use the application, and exploration questions for use with the Java applet.
Loan Calculator:	This virtual manipulative allows the user to explore scenarios of a loan repayment by manipulating the amount of the loan, interest rate, payment amount, frequency of payments, and length of the loan in years.

Lesson Plan

Name	Description
Determining the Hubble Constant:	Students will graph distance/velocity data of real galaxies to arrive at their own value of the Hubble constant (H). Once they have calculated their own value of H, they will use it to determine distances to real galaxies with known recessional velocities.
Functions and Everyday Situations:	This lesson unit is intended to help you assess how well students are able to articulate verbally the relationships between variables arising in everyday contexts, translate between everyday situations and sketch graphs of relationships between variables, interpret algebraic functions in terms of the contexts in which they arise and reflect on the domains of everyday functions and in particular whether they should be discrete or continuous.
Piles of Paper:	Piles of Paper is a student activity that demonstrates linear and exponential growth using heights of flat and folded paper. Data tables are created and then algebraic models are developed. Real world types of linear and exponential growth are also introduced.
The Towers of Hanoi: Experiential Recursive Thinking:	This lesson is about the Towers of Hanoi problem, a classic famous problem involving recursive thinking to reduce what appears to be a very large and difficult problem into a series of simpler ones. The learning objective is for students to begin to understand recursive logic and thinking, relevant to computer scientists, mathematicians and engineers. The lesson is experiential, in that each student will be working with her/his own Towers of Hanoi manipulative, inexpensively obtained. There is no formal prerequisite, although some familiarity with set theory and functions is helpful. The last three sections of the lesson involve some more formal concepts with recursive equations and proof by induction, so the students who work on those sections should probably be level 11 or 12 in a K-12 educational system. The lesson has a Stop Point for 50-minute classes, followed by three more segments that may require a half to full additional class time. So the teacher may use only those segments up to the Stop Point, or if two class sessions are to be devoted to the lesson, the entire set of segments. Supplies are modest, and may be a set of coins or some washers from a hardware store to assemble small piles of disks in front of each student, each set of disks representing a Towers of Hanoi manipulative. Or the students may assemble before the class a more complete Towers of Hanoi at home, as demonstrated in the video. The classroom activities involve attempting to solve with hand and mind the Towers of Hanoi problem and discussing with fellow students patterns in the process and strategies for solution.

Unit/Lesson Sequence

Name	Description
Direct and Inverse Variation:	"Lesson 1 of two lessons teaches students about direct variation by allowing them to explore a simulated oil spill using toilet paper tissues (to represent land) and drops of vegetable oil (to simulate a volume of oil). Lesson 2 teaches students about inverse variation by exploring the relationship between the heights of a fixed amount of water poured into cylindrical containers of different sizes as compared to the area of the containers' bases." from Insights into Algebra 1 - Annenberg Foundation.
	This sample Algebra 1 CMAP is a fully customizable resource and curriculum-planning tool that provides a framework for the Algebra 1 Course. The units and standards are customizable and the CMAP allows instructors to add lessons, worksheets, and other resources as needed. This CMAP also includes rows that automatically filter and display Math Formative Assessments System tasks, E-Learning Original Student Tutorials and Perspectives Videos that are aligned to the standards, available on CPALMS. Learn more about the sample Algebra 1 CMAP, its features and customizability by watching the following video: 

Sample Algebra 1 Curriculum Plan Using CMAP:	Could not load plugins: File not found **Using this CMAP** To view an introduction on the CMAP tool, please click here. To view the CMAP, click on the "Open Resource Page" button above; be sure you are logged in to your iCPALMS account. To use this CMAP, click on the "Clone" button once the CMAP opens in the "Open Resource Page." Once the CMAP is cloned, you will be able to see it as a class inside your iCPALMS My Planner (CMAPs) app. To access your My Planner App and the cloned CMAP, click on the iCPALMS tab in the top menu. All CMAP tutorials can be found within the iCPALMS Planner App or at the following URL: http://www.cpalms.org/support/tutorials_and_informational_videos.aspx

Formative Assessment

Name	Description
Functions From Graphs:	Students are asked to write a function given its graph.
The Cost of Water:	Students are asked to write a function to model the relationship between two variables described in a real-world context.
What Is the Function Rule?:	Students are asked to write function rules for sequences given tables of values.
Writing a Function From Ordered Pairs:	Students are given a table of values and are asked to write a linear function.
Writing an Exponential Function From a Description:	Students are asked to write an exponential function from a written description of an exponential relationship.
Writing an Exponential Function From a Table:	Students are asked to write an exponential function represented by a table of values.
Writing an Exponential Function From Its Graph:	Students are asked to write an exponential function given its graph.

Assessment

Name	Description
Sample 2 - High School Algebra 1 State Interim Assessment:	This is a State Interim Assessment for 9th-12th grades.
Sample 4 - High School Algebra 1 State Interim Assessment:	This is a State Interim Assessment for 9th-12th grades.

Student Resources

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Carbon 14 Dating in Practice II:	This problem introduces the method used by scientists to date certain organic material. It is based not on the amount of the Carbon 14 isotope remaining in the sample but rather on the ratio of Carbon 14 to Carbon 12. This ratio decreases, hypothetically, at a constant exponential rate as soon as the organic material has ceased to absorb Carbon 14, that is, as soon as it dies.
Creating Exponential Functions:	By the end of this tutorial, you should be able construct an exponential function from a graph, from a table of values, and from a description of a relationship in the real world.
Data Flyer:	Using this virtual manipulative, students are able to graph a function and a set of ordered pairs on the same coordinate plane. The constants, coefficients, and exponents can be adjusted using slider bars, so the student can explore the affect on the graph as the function parameters are changed. Students can also examine the deviation of the data from the function. This activity includes supplemental materials, including background information about the topics covered, a description of how to use the application, and exploration questions for use with the java applet.

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Parent Resources

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