Standard #: MAFS.912.A-SSE.2.3

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Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Grade: 912

Cluster: Write expressions in equivalent forms to solve problems. (Algebra 1 - Supporting Cluster) (Algebra 2 - Major Cluster) -
Clusters should not be sorted from Major to Supporting and then taught in that order. To do so would strip the coherence of the mathematical ideas and miss the opportunity to enhance the major work of the grade with the supporting clusters.

Date Adopted or Revised: 02/14

Content Complexity Rating: Level 2: Basic Application of Skills & Concepts - More Information

Date of Last Rating: 02/14

Status: State Board Approved

Assessed: Yes

TEST ITEM SPECIFICATIONS

Item Type(s): This benchmark may be assessed using: SHT , TI item(s)

Also assesses:
MAFS.912.A-SSEE.1.1

MAFS.912.A-SSE.1.2
Assessment Limits :
Items that require the student to transform a quadratic equation to vertex form, b/a must be an even integer.

For A-SSE.1.1, items should not ask the student to interpret zeros, the vertex, or axis of symmetry when the quadratic expression is in the form $ax^2 + bx + c$ (see F-IF.3.8).
For A-SSE.2.3c and A-SSE.1.1, exponential expressions are limited to simple growth and decay. If the number e is used then its approximate value should be given in the stem.

For A-SSE.2.3a and A-SSE.1.1, quadratic expressions should be univariate.

For A-SSE.2.3b, items should only ask the student to interpret the yvalue of the vertex within a real-world context.

For A-SSE.2.3, items should require the student to choose how to rewrite the expression.

In items that require the student to write equivalent expressions by factoring, the given expression may

- have integral common factors
- be a difference of two squares up to a degree of 4
- be a quadratic, $ax^2 + bx + c$, where $a > 0$ and a, b, and c are integers
- be a polynomial of four terms with a leading coefficient of 1 and highest degree of 3.

Calculator :
Neutral

Clarification :
Students will use equivalent forms of a quadratic expression to interpret the expression's terms, factors, zeros, maximum, minimum,

coefficients, or parts in terms of the real-world situation the expression represents.
Students will use equivalent forms of an exponential expression to interpret the expression's terms, factors, coefficients, or parts in terms of the real-world situation the expression represents.

Students will rewrite algebraic expressions in different equivalent forms by recognizing the expression's structure.

Students will rewrite algebraic expressions in different equivalent forms using factoring techniques (e.g., common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely) or simplifying expressions (e.g., combining like terms, using the distributive property, and other operations with polynomials).

Stimulus Attributes :
Items assessing A-SSE.2.3 and A-SSE.1.1 must be set in a real-world context.

Items that require an equivalent expression found by factoring may be in a real-world or mathematical context.

Items should contain expressions only.

Items may require the student to provide the answer in a specific form.

Response Attributes :
Items may require the student to choose an appropriate level of accuracy.

Items may require the student to choose and interpret units.

For A-SSE.1.1 and A-SSE.2.3, items may require the student to apply the basic modeling cycle.

SAMPLE TEST ITEMS (1)

Test Item #: Sample Item 1
Question:
Sue removes the plug from a trough to drain the water inside. The volume, in gallons, in the trough after it has been unplugged can be modeled by $4t^2-32t+63$, where t is time, in minutes.

A. Click on the correct property that will give Sue the amount of time it takes the trough to drain.

B. Click on the expression that will reveal the property.

Difficulty: N/A
Type: SHT: Selectable Hot Text

Related Courses

Course Number	Course Title
1200310:	Algebra 1 (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200320:	Algebra 1 Honors (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200330:	Algebra 2 (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200340:	Algebra 2 Honors (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200380:	Algebra 1-B (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200400:	Intensive Mathematics (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1207310:	Liberal Arts Mathematics (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200410:	Mathematics for College Success (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200700:	Mathematics for College Readiness (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
7912060:	Access Informal Geometry (Specifically in versions: 2014 - 2015 (course terminated))

7912070:	Access Liberal Arts Mathematics (Specifically in versions: 2014 - 2015, 2015 - 2018, 2018 - 2019, 2019 and beyond (current))
7912080:	Access Algebra 1A (Specifically in versions: 2014 - 2015, 2015 - 2018, 2018 - 2019, 2019 and beyond (current))
7912090:	Access Algebra 1B (Specifically in versions: 2014 - 2015, 2015 - 2018, 2018 - 2019, 2019 and beyond (current))
1200315:	Algebra 1 for Credit Recovery (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
1200335:	Algebra 2 for Credit Recovery (Specifically in versions: 2014 - 2015, 2015 - 2019 (course terminated))
1200385:	Algebra 1-B for Credit Recovery (Specifically in versions: 2014 - 2015, 2015 and beyond (current))
7912065:	Access Geometry (Specifically in versions: 2015 and beyond (current))
7912075:	Access Algebra 1 (Specifically in versions: 2014 - 2015, 2015 - 2018, 2018 - 2019, 2019 and beyond (current))

Related Access Points

Access Point

Access Points Number	Access Points Title
MAFS.912.A-SSE.2.AP.3b:	Given a quadratic function, explain the meaning of the zeros of the function (e.g., if f(x) = (x - c) (x - a) then f(a) = 0 and f(c) = 0).
MAFS.912.A-SSE.2.AP.3c:	Given a quadratic expression, explain the meaning of the zeros graphically (e.g., for an expression (x - a) (x - c), a and c correspond to the x-intercepts (if a and c are real).
MAFS.912.A-SSE.2.AP.3d:	Write expressions in equivalent forms by completing the square to convey the vertex form, to find the maximum or minimum value of a quadratic function, and to explain the meaning of the vertex.
MAFS.912.A-SSE.2.AP.3e:	Use properties of exponents (such as power of a power, product of powers, power of a product, and rational exponents, etc.) to write an equivalent form of an exponential function to reveal and explain specific information about its approximate rate of growth or decay.
MAFS.912.A-SSE.2.AP.4a:	Use the formula for the sum of finite geometric series to solve problems. $a\left(\frac{1 - r^n}{1 - r}\right)$

Related Resources

Problem-Solving Task

Name	Description
Building a General Quadratic Function:	In this resource, a method of deriving the quadratic formula from a theoretical standpoint is demonstrated. This task is for instructional purposes only and builds on "Building an explicit quadratic function."
Forms of Exponential Expressions:	There are many different ways to write exponential expressions that describe the same quantity, in this task the amount of a radioactive substance after *t* years. Depending on what aspect of the context we need to investigate, one expression of the quantity may be more useful than another. This task contrasts the usefulness of four equivalent expressions. Students first have to confirm that the given expressions for the radioactive substance are equivalent. Then they have to explain the significance of each expression in the context of the situation.
Graphs of Quadratic Functions:	Students compare graphs of different quadratic functions, then produce equations of their own to satisfy given conditions. This exploration can be done in class near the beginning of a unit on graphing parabolas. Students need to be familiar with intercepts, and need to know what the vertex is. It is effective after students have graphed parabolas in vertex form ($y=a(x-h)^2+k$), but have not yet explored graphing other forms.
Ice Cream:	This task illustrates the process of rearranging the terms of an expression to reveal different aspects about the quantity it represents, precisely the language being used in standard MAFS.912.A-SSE.2.3. Students are provided with an expression giving the temperature of a container at a time *t*, and have to use simple inequalities (e.g., that $2^t>0$ for all *t*) to reduce the complexity of an expression to a form where bounds on the temperature of a container of ice cream are made apparent.
Increasing or Decreasing? Variation 2:	The purpose of this task is to help students see manipulation of expressions as an activity undertaken for a purpose. Variation 1 of this task presents a related more complex expression already in the correct form to answer the question. The expression arises in physics as the reciprocal of the combined resistance of two resistors in parallel. However, the context is not explicitly considered here.
Profit of a Company:	This task compares the usefulness of different forms of a quadratic expression. Students have to choose which form most easily provides information about the maximum value, the zeros and the vertical intercept of a quadratic expression in the context of a real world situation. Rather than just manipulating one form into the other, students can make sense out of the structure of the expressions. (From *Algebra: Form and Function*, McCallum et al., Wiley 2010)
Seeing Dots:	The purpose of this task is to identify the structure in the two algebraic expressions by interpreting them in terms of a geometric context. Students will have likely seen this type of process before, so the principal source of challenge in this task is to encourage a multitude and variety of approaches, both in terms of the geometric argument and in terms of the algebraic manipulation.

Formative Assessment

Name	Description
College Costs:	Students are asked to transform an exponential expression so that the rate of change corresponds to a different time interval.
Jumping Dolphin:	Students are asked to find the zeros of a quadratic function in the context of a modeling problem.
Population Drop:	Students are asked to use the properties of exponents to show that two expressions are equivalent and compare the two functions in terms of what each reveals.
Rocket Town:	Students are asked to rewrite a quadratic expression in vertex form to find maximum and minimum values.

Original Student Tutorial

Name	Description
Finding the Maximum or Minimum of a Quadratic Function:	Learn to complete the square of a quadratic expression and identify the maximum or minimum value of the quadratic function it defines. In this interactive tutorial, you'll also interpret the meaning of the maximum and minimum of a quadratic function in a real world context.
Finding the Zeros of Quadratic Functions:	Quadratic functions can be used to model real-world phenomena. Key features of quadratic functions such as maximum values and zeros can often reveal important qualities of these phenomena. By the end of this tutorial, you should be able to find the zeros of a quadratic function and interpret their meaning in real-world contexts.

Lesson Plan

Name	Description
Forming Quadratics:	This lesson unit is intended to help you assess how well students are able to understand what the different algebraic forms of a quadratic function reveal about the properties of its graphical representation. In particular, the lesson will help you identify and help students who have the following difficulties in understanding how the factored form of the function can identify a graph's roots, how the completed square form of the function can identify a graph's maximum or minimum point, and how the standard form of the function can identify a graph's intercept.
Hip to be (completing the) Square:	This lesson is an introduction to completing the square and focuses on MAFS.912.A-SSE.2.3b. Students will learn what it means to "complete the square" with a quadratic trinomial expression. They will practice both with manipulatives and mathematically, and will then use that information to find the maximum or minimum value of an expression using the vertex form of a quadratic. This lesson moves through all levels of Concrete, Representational, Abstract instruction.
Modeling Conditional Probabilities 2:	This lesson unit is intended to help you assess how well students understand conditional probability, and, in particular, to help you identify and assist students who have the following difficulties representing events as a subset of a sample space using tables and tree diagrams and understanding when conditional probabilities are equal for particular and general situations.
Sorting Equations and Identities:	This lesson is intended to help you assess how well students are able to: • Recognize the differences between equations and identities. • Substitute numbers into algebraic statements in order to test their validity in special cases. • Resist common errors when manipulating expressions such as $2(x - 3) = 2x - 3$; $(x + 3)^2 = x^2 + 3^2$. • Carry out correct algebraic manipulations. It also aims to encourage discussion on some common misconceptions about algebra.
Using algebra tiles and tables to factor trinomials (less guess and check!):	Students will use algebra tiles to visually see how to factor trinomials. In addition, they will use a 3 x 3 table. This process makes students more confident when factoring because there is less guess and check involved in solving each problem.

Tutorial

Name	Description
Power of a Power Property:	This tutorial demonstrates how to use the power of a power property with both numerals and variables.

Assessment

Name	Description
Sample 1 - High School Algebra 1 State Interim Assessment:	This is the State Interim Assessment for high school.
Sample 1 - High School Algebra 2 State Interim Assessment:	This is a State Interim Assessment for 9th-12th grades.
Sample 2 - High School Algebra 1 State Interim Assessment:	This is a State Interim Assessment for 9th-12th grades.
Sample 2 - High School Algebra 2 State Interim Assessment:	This is a State Interim Assessment for 9th-12th grades.
Sample 3 - High School Algebra 1 State Interim Assessment:	This is a State Interim Assessment for 9th-12th grades.
Sample 3 - High School Algebra 2 State Interim	This is a State Interim Assessment for 9th-12th grades.

Assessment:	

Unit/Lesson Sequence

Name	Description
Sample Algebra 1 Curriculum Plan Using CMAP:	This sample Algebra 1 CMAP is a fully customizable resource and curriculum-planning tool that provides a framework for the Algebra 1 Course. The units and standards are customizable and the CMAP allows instructors to add lessons, worksheets, and other resources as needed. This CMAP also includes rows that automatically filter and display Math Formative Assessments System tasks, E-Learning Original Student Tutorials and Perspectives Videos that are aligned to the standards, available on CPALMS. Learn more about the sample Algebra 1 CMAP, its features and customizability by watching the following video: Using this CMAP To view an introduction on the CMAP tool, please click here. To view the CMAP, click on the "Open Resource Page" button above; be sure you are logged in to your iCPALMS account. To use this CMAP, click on the "Clone" button once the CMAP opens in the "Open Resource Page." Once the CMAP is cloned, you will be able to see it as a class inside your iCPALMS My Planner (CMAPs) app. To access your My Planner App and the cloned CMAP, click on the iCPALMS tab in the top menu. All CMAP tutorials can be found within the iCPALMS Planner App or at the following URL: http://www.cpalms.org/support/tutorials_and_informational_videos.aspx

Student Resources

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Building a General Quadratic Function:	In this resource, a method of deriving the quadratic formula from a theoretical standpoint is demonstrated. This task is for instructional purposes only and builds on "Building an explicit quadratic function."
Finding the Maximum or Minimum of a Quadratic Function:	Learn to complete the square of a quadratic expression and identify the maximum or minimum value of the quadratic function it defines. In this interactive tutorial, you'll also interpret the meaning of the maximum and minimum of a quadratic function in a real world context.
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	The purpose of this task is to help students see manipulation of expressions as an activity undertaken for a purpose.

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