Standard 2 : Differential Calculus

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Develop an understanding of the derivative as an instantaneous rate of change, using geometrical, numerical, and analytical methods. Use this definition to find derivatives of algebraic and transcendental functions and combinations of these functions (using, for example, sums, composites, and inverses). Find second and higher order derivatives. Understand and use the relationship between differentiability and continuity. Understand and apply the Mean Value Theorem. Find derivatives of algebraic, trigonometric, logarithmic, and exponential functions. Find derivatives of sums, products, and quotients, and composite and inverse functions. Find derivatives of higher order, and use logarithmic differentiation and the Mean Value Theorem.

General Information

Number: MAFS.912.C.2

Title: Differential Calculus

Type: Cluster

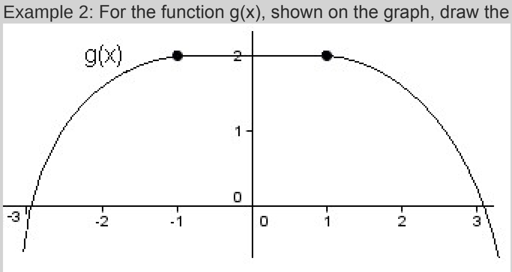
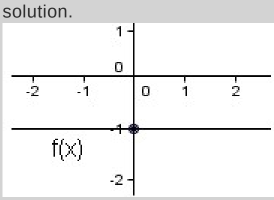
Subject: Mathematics

Grade: 912

Domain-Subdomain: Calculus

Related Standards

This cluster includes the following benchmarks

Code	Description
MAFS.912.C.2.1	Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as an instantaneous rate of change or as the slope of the tangent line. **Clarifications:** Example: Approximate the derivative of $f(x) = x^2$ at x=5 by calculating values of $\frac{f(x+h)-f(x)}{h}$ for values of h that are very close to zero. Use a diagram to explain what you are doing and what the result means.
MAFS.912.C.2.2	State, understand, and apply the definition of derivative. **Clarifications:** Example 1 (related to the example given in C.2.1):Find $\lim_{h\to 0}\frac{(5+h)^2-5^2}{h}$. What does the result tell you? Use the limit given above to determine the derivative function for f(x). In other words calculate f '(x) = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ for $f(x) = x^2$. Example 2: For the function g(x), shown on the graph, draw the graph of g'(x) by estimation. Explain how you arrived at your solution.  Example 3: The graph of the function f(x) is given below. Find a function g(x) such that the derivative of g(x) will be f(x). Explain your solution. 
MAFS.912.C.2.3	Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions. **Clarifications:** Example 1: Find $\frac{dy}{dx}$ for the function $y = x^2$. Example 2: Find $\frac{dy}{dx}$ for the function y = ln(x).

	dx
MAFS.912.C.2.4	Find the derivatives of sums, products, and quotients. **Clarifications:** Example 1: Find the derivative of the function f(x)= xcos(x). Example 2: Using the quotient rule for derivatives, show that the derivative of f(x) = tan(x) is f '(x) = $\sec^2(x)$.
MAFS.912.C.2.5	Find the derivatives of composite functions using the Chain Rule. **Clarifications:** Example 1: Find $f'(x)$ for $f(x) = (x^2 + 2)^2$. Example 2: Find $f'(x)$ for $f(x) = \sin(\frac{1}{x})$.
MAFS.912.C.2.6	Find the derivatives of implicitly-defined functions. **Clarifications:** Example: For the equation $xy - x^2y^2 = 5$, find $\frac{dy}{dx}$ at the point (2, 3).
MAFS.912.C.2.7	Find derivatives of inverse functions. **Clarifications:** Example: Let $f(x) = 2x^3$ and $g(x) = f^{-1}(x)$ find $g'(2)$.
MAFS.912.C.2.8	Find second derivatives and derivatives of higher order. **Clarifications:** Example: Let $f(x) = e^{5x}$. Find $f''(x)$ and $f'''(x)$.
MAFS.912.C.2.9	Find derivatives using logarithmic differentiation. **Clarifications:** Example 1: Find $\frac{dy}{dx}$ for the following equation: $y = \sqrt{(x+3)^3(x-7)}$. Example 2: Find the derivative of $f(x) = (3x^2 + 5)^x$.
MAFS.912.C.2.10	Understand and use the relationship between differentiability and continuity. **Clarifications:** Example 1: Let f(x)=1/x. Is f(x) continuous at x = 0? Is f(x) differentiable at x = 0? Explain your answers. Example 2: Is f(x) = IxI continuous at x=0? Is f(x) differentiable at x=0? Explain your answers.
MAFS.912.C.2.11	Understand and apply the Mean Value Theorem. **Clarifications:** Example 1: Let $f(x) = \sqrt{x}$. On the interval [1, 9], find the value of c such that $\frac{f(9) - f(1)}{9 - 1} = f'(c)$. Example 2: At a car race, two cars join the race at the same point at the same time. They finish the race in a tie. Prove that some time during the race, the two cars had exactly the same speed. (Hint: Define f(t), g(t), and h(t), where f(t) is the distance that car 1 has traveled at time t, g(t) is the distance that car 2 has travelled at time t, and h(t) = f(t) - g(t).)

Related Resources

Vetted resources educators can use to teach the concepts and skills in this topic.

Tutorials

Name	Description
Calculus: Derivatives 1:	In this video we will learn that a derivative is simply the slope of a curve at any given point.
Calculating Slope of Tangent Line Using Derivative Definition:	In this video we will use find the slope of the tangent line to determine the derivative.
Mean Value Theorem:	We will learn and apply the Mean Value Theorem.
Derivative as Slope of a Tangent Line:	We will find the derivative of a function by finding the slope of the tangent line.
Mean Value Theorem:	In this video we will take an in depth look at the Mean Value Theorem.
The Derivative of f(x)=x^2 for Any x:	In this video we will find the derivative of a function based on the slope of the tangent line.
Using the Product Rule and the Chain Rule:	In this video we will use the chain rule and the product rule together to find a derivative of a composite function.
The Product Rule for Derivatives:	In this video will will apply the product rule to find the derivative of two functions.
Product Rule for More Than Two Functions:	In this video, we will use the product rule to find the derivative of the product of three functions.

Derivative of Log with Arbitrary Base:	In this video, we will find the derivative of a log with an arbitrary base.
Chain Rule for Derivative of 2^x:	Here we will see how the chain rule is used to find the derivative of a logarithmic function.
Chain Rule Introduction:	This video is an introduction on how to apply the chain rule to find the derivative of a composite function.
Chain Rule Definition and Example:	In this video we will define the chain rule and use it to find the derivative of a function.
Chain Rule With Triple Composition:	We will use the chain rule to find the derivative of a triple-composite function.
Chain Rule Example Using Visual Information:	In this video we will analyze the graph of a function and its tangent line, then use the chain rule to find the value.
Chain Rule Example Using Visual Function Definitions:	We will use the chain rule to find the value of a composite function at a given point.

Video/Audio/Animation

Name	Description
MIT BLOSSOMS - The Physics of Boomerangs:	This learning video explores the mysterious physics behind boomerangs and other rapidly spinning objects. Students will get to make and throw their own boomerangs between video segments! A key idea presented is how torque causes the precession of angular momentum. One class period is required to complete this learning video, and the optimal prerequisites are a familiarity with forces, Newton's laws, vectors and time derivatives. Each student would need the following materials for boomerang construction: cardboard (roughly the size of a postcard), ruler, pencil/pen, scissors, protractor, and a stapler.

Virtual Manipulative

Name	Description
Derivative Plotter:	This online applet that depicts the derivative of a given function. Can use demo examples or a user-defined function.

Student Resources

Vetted resources students can use to learn the concepts and skills in this topic.

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